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THE FARRELL–JONES CONJECTURE IN ALGEBRAIC K- AND L-THEORY

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One of my favorite conjectures in geometric topology is the *Borel Conjecture*. It says that a homotopy equivalence between closed aspherical topological manifolds is homotopic to a homeomorphism. This implies that two closed aspherical topological manifolds are homeomorphic if and only if their fundamental groups are isomorphic. (This is not true if one considers smooth manifolds and replaces homeomorphic by diffeomorphic.) This is the topological analogue of Mostow rigidity.

Another prominent conjecture is the *Novikov Conjecture*. It asserts that the higher signatures of a closed oriented smooth manifold are homotopy invariants. It is motivated by the signature formula of Hirzebruch.

Furthermore the *Bass Conjecture* has gotten a lot of attention since it was formulated. It says that for a group G and an integral domain R of characteristic 0 the *Hattori–Stallings rank* of a finitely generated projective RG-module evaluated at an element g of G is non-trivial only if g has finite order $|g|$ and $|g| \cdot 1_R$ is not a unit in R.

Finally we mention the conjectures that for a torsionfree group G the *reduced projective class group* $\widetilde{K}_0(\mathbf{Z}G)$ and its *Whitehead group* $\mathrm{Wh}(G)$ vanish. The last two conjectures have equivalent geometric counterparts, namely: if the fundamental group G is finitely presented and torsionfree, then any *finitely dominated CW-complex* is homotopy equivalent to a finite CW-complex and any *h-cobordism* over a closed manifold of dimension ≥ 5 is trivial.

Also a lot of work was done about the *Kaplansky Conjecture* which predicts for an integral domain R and a torsionfree group G that 0 and 1 are the only idempotents in the group ring RG.

It turns out the Farrell–Jones Conjecture for algebraic K- and L-theory does imply all of the conjectures above and gives a good understanding of the algebraic K- and L-theory of group rings RG in terms of the algebraic K- and L-theory of the coefficient ring R and the homology of the group G. I will formulate it only for a torsionfree group G and a regular ring R. In this case the *Farrell–Jones Conjecture* predicts that the classical assembly maps

$$\begin{aligned} H_n(BG;\mathbf{K}_R) &\xrightarrow{\cong} K_n(RG)\,; \\ H_n(BG;\mathbf{L}_R) &\xrightarrow{\cong} L_n(RG)\,; \end{aligned}$$

are bijective for all n. The sources of the assembly maps above are homology theories such that $H_n(\mathrm{pt.};\mathbf{K}_R) = K_n(R)$ and $H_n(\mathrm{pt.};\mathbf{L}_R) = L_n(R)$ hold. In the L-theory case one does not need regular. The general formulation uses more elaborate *equivariant homology theories* and *classifying spaces of families*.

The original formulation of the *Farrell–Jones Conjecture* can be found in [3], 1.6 on page 257. The Farrell–Jones Conjecture is still open but known for a good class of groups. Its non-commutative counterpart is the *Baum–Connes Conjecture*. For more explanations, information about the status and references concerning the Farrell–Jones Conjecture we refer for instance to the survey article [4]. For very recent new developments we refer to [1] and [2].

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