On the Universal Space for Group Actions with Compact Isotropy

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Abstract

Let G be a locally compact topological group **and $\underline{E}G$ its universal space for the family of compact subgroups.** We give criteria for this space to **be G-homotopy equivalent to a d-dimensional G-CW-complex, a finite G-CW-complex or a G-CW-complex of finite type.** Essentially we reduce these questions to discrete groups, and to the homological algebra of the orbit category of discrete groups with respect to certain families of subgroups.

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Introduction

Throughout this paper we denote by G a locally compact topological group (where locally compact always includes Hausdorff). We denote by G_0 the *component of the identity element* and by $\overline{G} = G/G_0$ its *component group*. Notice that G_0 is locally compact and connected and $\overline{G}$ is locally compact and *totally disconnected*, i.e. each component consists of exactly one point. Subgroup will always mean closed subgroup. Sometimes we make the additional assumption

(S) For any closed subgroup $H \subset G$ the projection $p : G \longrightarrow G/H$ has a local cross section, i.e. there is a neighborhood U of eH together with a map $s : U \to G$ satisfying $p \circ s = \mathrm{id}_U$.

Condition (S) is automatically satisfied if G is discrete, if G is a Lie group, or more generally, if G is locally compact and second countable and has finite covering dimension [15]. The metric needed in [15] follows under our assumptions, since a locally compact Hausdorff space is regular and regularity in a second countable space implies metrizability.

A *family* $\mathcal{F}$ consists of a set of (closed) subgroups of G with the property that for any $H, K \in \mathcal{F}$ and $g \in G$, the subgroups $g^{-1}Hg$ and $H \cap K$ belong to $\mathcal{F}$. Notice that we do not require that $\mathcal{F}$ is closed under taking subgroups. A ***universal space of*** G

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for the family $\mathcal{F}$ is a G-CW-complex $E(G,\mathcal{F})$ such that the fixed point set $E(G,\mathcal{F})^H$ is weakly contractible for $H \in \mathcal{F}$ and all its isotropy groups belong to $\mathcal{F}$. Recall that a map $f : X \to Y$ of spaces is a *weak homotopy equivalence* if and only if the induced map $f_* : \pi_n(X,x) \to \pi_n(Y,f(x))$ is an isomorphism for all base points $x \in X$ and for all $n \geq 0$ and that a space X is *weakly contractible* if and only if the projection $X \to \{*\}$ onto the space consisting of one point is a weak homotopy equivalence. The G-CW-complex $E(G,\mathcal{F})$ has the universal property that for any G-CW-complex X whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $X \longrightarrow E(G,\mathcal{F})$. In particular, $E(G,\mathcal{F})$ is unique up to G-homotopy. The notion of a G-CW-complex can be found for instance in [10, Definition 1.2], the existence of $E(G,\mathcal{F})$ follows for instance from [10, 2.2], and the universal property of $E(G,\mathcal{F})$ is a consequence of the Equivariant Whitehead Theorem [10, Theorem 2.4]. If $\mathcal{F}$ is the family of compact subgroups $\mathcal{COM}$, we will often abbreviate $E(G,\mathcal{COM})$ by $\underline{E}G$. Notice that for a discrete group $\mathcal{COM}$ is the same as the family $\mathcal{FIN}$ of finite subgroups.

In Section 1 we will explain **our notion of** $E(G,\mathcal{F})$ and **compare it** with the similar notion due to tom Dieck [4, section I.6]. We mention that these spaces $E(G,\mathcal{F})$ and in particular $\underline{E}G$ play an important role in the formulation of the Baum-Connes Conjecture [3, Conjecture 3.15 on p. 254], the Isomorphism Conjecture in algebraic K- and L-theory of Farrell and Jones [5], the generalization of the completion theorem of Atiyah and Segal for finite groups to infinite discrete groups [12] and in the construction of classifying spaces for equivariant bundles [4, Section I.8 and I.9]. More information about models for $\underline{E}G$ can be found for instance in [3].

We call any G-CW-complex in the G-homotopy class of $E(G,\mathcal{F})$ a *G-CW-model for* $E(G,\mathcal{F})$. In this paper, we investigate the *type of* $E(G,\mathcal{F})$, i.e. whether there is an m-dimensional G-CW-model, a finite G-CW-model or a G-CW-model of finite type for $E(G,\mathcal{F})$. A G-CW-complex X is *finite* if it is built by finitely many equivariant cells or, equivalently, if $G\backslash X$ is compact. It is called *of finite type* if each skeleton X_n is finite. For discrete groups the type of $\underline{E}G$ has been investigated in [9], [11] and [16]. In Section 2, we will give a necessary and sufficient algebraic criterion which not only applies to $\mathcal{FIN}$ but to any family $\mathcal{F}$. Namely, in Section 2 we will explain and prove

Theorem 0.1 *Let G be a discrete group and let $d \geq 3$. Then we have:*

(a) There is a d-dimensional G-CW-model for $E(G,\mathcal{F})$ if and only if the constant $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module $\underline{\mathbb{Z}}$ has a d-dimensional projective resolution;

(b) There is a G-CW-model for $E(G,\mathcal{F})$ of finite type if and only if $E(G,\mathcal{F})$ has a G-CW-model with finite 2-skeleton and the constant $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module $\underline{\mathbb{Z}}$ has a projective resolution of finite type;

(c) There is a finite G-CW-model for $E(G,\mathcal{F})$ if and only if $E(G,\mathcal{F})$ has a G-CW-model with finite 2-skeleton and the constant $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module $\underline{\mathbb{Z}}$ has a finite free resolution over $Or(G,\mathcal{F})$;

(d) There is a G-CW-model with finite 2-skeleton for $\underline{E}G = E(G,\mathcal{FIN})$ if and only if there are only finitely many conjugacy classes of finite subgroups $H \subset G$ and for any finite subgroup $H \subset G$ its Weyl group $WH := NH/H$ is finitely presented.

In Section 3 we will reduce the case of a totally disconnected group to the one of a discrete group, **as summarized in Theorem 0.2**. Throughout the paper we will denote the *discretization* of a topological group G by G_d, i.e. the same group but now with the discrete topology. Given a family $\mathcal{F}$ of (closed) subgroups of G, denote by $\mathcal{F}_d$ the same set of subgroups, but now in connection with G_d. Notice that $\mathcal{F}_d$ is again a family.

Theorem 0.2 *Let G be a locally compact totally disconnected group and let $\mathcal{F}$ be a family of subgroups of G. Then there is a G-CW-model for $E(G, \mathcal{F})$ that is d-dimensional (resp. finite, resp. of finite type) if and only if there is a G_d-CW-model for $E(G_d, \mathcal{F}_d)$ that is d-dimensional (resp. finite, resp. of finite type).*

The case of an *almost connected* group G, i.e. $\overline{G}$ is compact, has already been treated by Abels [2, Corollary 4.14]. Namely, for an almost connected (locally compact) group G there is a model for $\underline{E}G$ consisting of one equivariant cell G/K. Notice that K is then necessarily a maximal compact subgroup of G and uniquely determined by this property up to conjugation. In Section 4 we use this result to reduce the case of a locally compact group G to a totally disconnected group. We show

Theorem 0.3 *Let G be a locally compact group satisfying (S) and let $\overline{G} := G/G_0$. Then there is a G-CW-model for $\underline{E}G$ that is d-dimensional (resp. finite, resp. of finite type) if and only if $\underline{E}\overline{G}$ has a $\overline{G}$-CW-model that is d-dimensional (resp. finite, resp. of finite type).*

If we combine Theorem 0.1, Theorem 0.2 and Theorem 0.3 we get

Theorem 0.4 *Let G be a locally compact group satisfying (S). Denote by $\overline{\mathcal{COM}}$ the family of compact subgroups of its component group $\overline{G}$ and let $d \geq 3$. Then*

(a) There is a d-dimensional G-CW-model for $\underline{E}G$ if and only if the constant $\mathbb{Z}\operatorname{Or}(\overline{G}_d, \overline{\mathcal{COM}}_d)$-module $\underline{\mathbb{Z}}$ has a d-dimensional projective resolution;

(b) There is a G-CW-model for $\underline{E}G$ of finite type if and only if $E(\overline{G}_d, \overline{\mathcal{COM}}_d)$ has a $\overline{G}_d$-CW-model with finite 2-skeleton and the constant $\mathbb{Z}\operatorname{Or}(\overline{G}_d, \overline{\mathcal{COM}}_d)$-module $\underline{\mathbb{Z}}$ has a projective resolution of finite type;

(c) There is a finite G-CW-model for $\underline{E}G$ if and only if $E(\overline{G}_d, \overline{\mathcal{COM}}_d)$ has a $\overline{G}_d$-CW-model with finite 2-skeleton and the constant $\mathbb{Z}\operatorname{Or}(\overline{G}_d, \overline{\mathcal{COM}}_d)$-module $\underline{\mathbb{Z}}$ has a finite free resolution.

In particular we see from Theorem 0.3 that, for a Lie group G, type questions about $\underline{E}G$ are equivalent to the corresponding type questions of $\underline{E}\pi_0(G)$, since $\pi_0(G) = \overline{G}$ is discrete (cf. [11, Problem 7.1]). In this case the family $\overline{\mathcal{COM}}_d$ appearing in Theorem 0.4 is just the family $\mathcal{FIN}$ of finite subgroups of $\pi_0(G)$.

1 Review of the Universal Space for a Family of Subgroups Comment: `Headline changed.`

Recall from the introduction the G-CW-complex $E(G,\mathcal{F})$. In particular, notice that we **only assume that the fixed point sets $E(G,\mathcal{F})^H$ for $H \in \mathcal{F}$ are weakly contractible, and not necessarily contractible.** If G is discrete, then each fixed point set $E(G,\mathcal{F})^H$ has the homotopy type of a CW-complex and is contractible for $H \in \mathcal{F}$. If $\overline{G}$ is discrete and $\mathcal{F} = \mathcal{COM}$, then $E(G,\mathcal{F})^H$ is contractible for $H \in \mathcal{COM}$ by Proposition 4.3. In general $E(G,\mathcal{F})^H$ need not be contractible as the following example shows.

Let G be totally disconnected and let $\mathcal{F}$ be the trivial family $\mathcal{TR}$ consisting of one element, namely the trivial group. We claim that then $E(G,\mathcal{TR})$ is contractible if and only if G is discrete. If G is discrete, we already know that $E(G,\mathcal{TR})$ is contractible. Suppose now that $E(G,\mathcal{TR})$ is contractible. We obtain a numerable G-principal bundle $G \to E(G,\mathcal{TR}) \to G\backslash E(G,\mathcal{TR})$ by the Slice Theorem [10, Theorem 1.37] and the fact that the quotient $G\backslash E(G,\mathcal{TR})$ is a CW-complex and hence paracompact. This implies that it is a fibration by a result of Hurewicz [17, Theorem on p. 33]. Since $E(G,\mathcal{TR})$ is contractible, G and the loop space $\Omega(G\backslash E(G,\mathcal{TR}))$ are homotopy equivalent [17, 6.9* on p. 137, 6.10* on p. 138, Corollary 7.27 on p. 40]. Since $G\backslash E(G,\mathcal{TR})$ is a CW-complex, $\Omega(G\backslash E(G,\mathcal{TR}))$ has the homotopy type of a CW-complex [13]. Let $f : G \to X$ be a homotopy equivalence from G to a CW-complex X. Then the induced map $\pi_0(G) \to \pi_0(X)$ between the set of path components is bijective. Hence any preimage of a path component of X is a point since G is totally disconnected. Since X is locally path-connected each path component of X is open in X. We conclude that G is the disjoint union of the preimages of the path components of X and each of these preimages is open in G and consists of one point. Hence G is discrete.

There is another variant of the **universal** space of a group G for a family $\mathcal{F}$ which we review next ([4, Theorem 6.6. on p. 47]. The assumption that G is a compact Lie group is not needed). We denote the space considered there by $J(G,\mathcal{F})$ since it is constructed by a variant of Milnor's infinite join construction. Namely, a model for $J(G,\mathcal{F})$ is $*_{n=1}^{\infty} Z$, where Z is a disjoint union of homogeneous spaces G/H_i such that each G/H_i is G-isomorphic to G/H for one $H \in \mathcal{F}$ and each $H \in \mathcal{F}$ occurs this way. This is an $\mathcal{F}$-numerable G-space with the universal property that for any $\mathcal{F}$-numerable G-space X there is up to homotopy precisely one G-map $X \to J(G,\mathcal{F})$. Again $J(G,\mathcal{F})$ is unique up to G-homotopy. In contrast to $E(G,\mathcal{F})$ the H-fixed point set $J(G,\mathcal{F})^H$ is always contractible for $H \in \mathcal{F}$. Since $E(G,\mathcal{F})$ is a G-CW-complex and hence an $\mathcal{F}$-numerable G-space, there is a G-map $f : E(G,\mathcal{F}) \to J(G,\mathcal{F})$ unique up to G-homotopy. Obviously f is a *weak G-homotopy equivalence*, i.e. $f^H : E(G,\mathcal{F})^H \to J(G,\mathcal{F})^H$ is a weak homotopy equivalence for each $H \subset G$. In other words, $E(G,\mathcal{F})$ is a G-CW-approximation of $J(G,\mathcal{F})$. We know that f cannot be a G-homotopy equivalence in general since $E(G,\mathcal{F})^H$ is not contractible in general. Hence these concepts are different. However, for any G-CW-complex X whose isotropy groups belong to $\mathcal{F}$, any G-map $X \to J(G,\mathcal{F})$ lifts uniquely up to G-homotopy over the G-map $f : E(G,\mathcal{F}) \to J(G,\mathcal{F})$. Moreover, if G is discrete or if G is a Lie group and $\mathcal{F}$ is contained in $\mathcal{COM}$, then $f : E(G,\mathcal{F}) \to J(G,\mathcal{F})$ is a G-homotopy equivalence and these concepts agree. This can be seen as follows.

Under the assumptions on G and $\mathcal{F}$, $*_{n=0}^{k} Z$ has the G-homotopy type of a G-CW-

complex and hence $*_{n=1}^{\infty} Z_{\text{weak}}$ has the G-homotopy type of a G-CW-complex, where $*_{n=1}^{\infty} Z_{\text{weak}}$ is equipped with the weak topology with respect to the filtration by the subspaces $*_{n=0}^{k} Z$ for $k = 1, 2, \ldots$. This follows for instance from [10, section 7]. (See also [8].) One checks that for a G-space X with a G-invariant G-covering and locally finite G-invariant subordinate partition of unity, the G-map $X \to J(G, \mathcal{F})$ constructed in [4, Lemma 6.13 on p. 49 and Lemma 6.9 on p. 48] actually factorizes through $*_{n=1}^{\infty} Z_{\text{weak}}$, since locally this map takes values in one of the subspaces $*_{n=0}^{k} Z$. In particular we obtain a G-map $J(G, \mathcal{F}) \to *_{n=1}^{\infty} Z_{\text{weak}}$. Since $*_{n=1}^{\infty} Z_{\text{weak}}$ is a G-CW-complex we obtain a G-map $h : J(G, \mathcal{F}) \to E(G, \mathcal{F})$. By the universal properties both compositions $h \circ f$ and $f \circ h$ are G-homotopic to the identity.

In the case $\mathcal{F} = \mathcal{COM}$ there is another model for the universal space of G, well known from harmonic analysis, described in [2, §2]. Denote by $C_0(G)$ the space of complex-valued continuous functions on G vanishing at infinity, endowed with the sup-norm-topology. By $g \cdot f(x) := f(g^{-1}x)$, $g \in G$, $f \in C_0(G)$, G acts isometrically on $C_0(G)$. Denote by $PC_0(G)$ the subspace of real-valued functions $f \neq 0$ that only take non-negative values. Then $PC_0(G)$ is a final object in the homotopy category of numerably proper G-spaces. As [2] and [4] work over the same category ([2, Prop. 3.9]), the models $PC_0(G)$ and $J(G, \mathcal{COM})$ are G-homotopy equivalent.

2 Discrete Groups

Throughout this section G denotes a discrete group. Finiteness conditions for $E(G, \mathcal{F})$ focussing on the family $\mathcal{F} = \mathcal{FIN}$ of finite subgroups have already been studied in [9], [11] and [16]. In this section we translate **type** questions about $E(G, \mathcal{F})$ for G and a family of subgroups $\mathcal{F}$ to homological algebra of modules over the associated orbit category $\operatorname{Or}(G, \mathcal{F})$. We begin by recalling some basic definitions.

The *orbit category* $\operatorname{Or}(G)$ of G is the small category whose objects are homogeneous G-spaces G/H and whose morphisms are G-maps. Let $\operatorname{Or}(G, \mathcal{F})$ be the full subcategory of $\operatorname{Or}(G)$ consisting of those objects G/H for which H belongs to $\mathcal{F}$. A $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$*-module* is a contravariant functor from $\operatorname{Or}(G, \mathcal{F})$ to the category of $\mathbb{Z}$-modules. A morphism of such modules is a natural transformation. The category of $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-modules inherits the structure of an abelian category from the standard structure of an abelian category on the category of $\mathbb{Z}$-modules. In particular the notion of a projective $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module is defined. The *free* $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module $\mathbb{Z}\operatorname{map}(G/?, G/K)$ *based at the object* G/K is the $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module that assigns to an object G/H the free $\mathbb{Z}$-module $\mathbb{Z}\operatorname{map}_G(G/H, G/K)$ generated by the set $\operatorname{map}_G(G/H, G/K)$. The key property of it is that for any $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module N there is a natural bijection of $\mathbb{Z}$-modules

$$\hom_{\mathbb{Z}\operatorname{Or}(G,\mathcal{F})}(\mathbb{Z}\operatorname{map}_G(G/?, G/K), N) \xrightarrow{\cong} N(G/K), \quad \phi \mapsto \phi(G/K)(\mathrm{id}_{G/K})$$

which is an application of the Yoneda Lemma. A $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module is *free* if it is isomorphic to a direct sum $\oplus_{i \in I} \mathbb{Z}\operatorname{map}(G/?, G/K_i)$ for appropriate choice of objects G/K_i and index set I. A $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module is called *finitely generated* if it is a quotient of a $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module of the shape $\oplus_{i \in I} \mathbb{Z}\operatorname{map}(G/?, G/K_i)$ with a finite index set I. Notice that a lot of standard facts for $\mathbb{Z}$-modules carry over to $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-modules. For instance, a $\mathbb{Z}\operatorname{Or}(G, \mathcal{F})$-module is projective or finitely generated projective respectively if and only if it is a direct summand in a free $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module or a finitely generated free $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module respectively. The notion of a *projective resolution* P_* of a $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module is obvious. We call P_* *of finite type* if each P_n is finitely generated projective. We call P_* *finite* if P_* is both of finite type and finite-dimensional. Each $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module has a projective resolution.

Definition 2.1 *Let G be a discrete group and (X,A) a relative G-CW-complex whose isotropy groups belong to the family $\mathcal{F}$. The contravariant functor*

$$\begin{array}{rcl} C^c_*(X,A) : \operatorname{Or}(G,\mathcal{F}) & \longrightarrow & \mathbb{Z}-\mathsf{Chain\ complexes} \\ G/H & \longmapsto & C^c_*(X^H,A^H) \end{array}$$

is called the cellular $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-chain complex of (X,A).

Functoriality comes from the fact that $X^H = \operatorname{map}_G(G/H,X)$. Notice that (X^H,A^H) is canonically a CW-complex, hence we can speak of its cellular chain complex $C^c_*(X^H,A^H)$. As in the nonequivariant situation the chain modules are free with basis given by the (equivariant) cells. Namely, we have

Lemma 2.2 *For any $n\in\mathbb{Z}$ the n-th chain module $C^c_n(X) : \operatorname{Or}(G,\mathcal{F}) \longrightarrow \mathbb{Z}-\mathsf{Modules}$ is a free $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-module.*

Proof: Let the n-skeleton of X be given by a pushout

$$\begin{tikzcd} \coprod_{I_n} G/H_i \times S^{n-1} \arrow[r] \arrow[d, hook, "\operatorname{id}\times i"'] & X_{n-1} \arrow[d] \\ \coprod_{I_n} G/H_i \times D^n \arrow[r] & X_n. \end{tikzcd}$$

Since $\operatorname{id}\times i$ is a cofibration, we get by excision in G/H natural isomorphisms

$$\begin{array}{rl} C^c_n(X,A)(G/H) & \cong\ H_n(X^H_n,X^H_{n-1}) \ \cong\ H_n(\coprod_{I_n}(G/H_i)^H\times(D^n,S^{n-1})) \\ \cong\ \bigoplus_{I_n} H_n((G/H_i)^H\times(D^n,S^{n-1})) & \cong\ \bigoplus_{I_n} H_0((G/H_i)^H) \ \cong\ \bigoplus_{I_n}\mathbb{Z}[(G/H_i)^H] \\ & \cong\ \bigoplus_{I_n}\mathbb{Z}[\operatorname{map}_G(G/H,G/H_i)]. \quad \blacksquare \end{array} \tag{2.3}$$

Corollary 2.4 *Let X be a G-CW-complex whose isotropy groups belong to the family $\mathcal{F}$. Then X is d-dimensional (resp. finite, resp. of finite type) if and only if its cellular $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-chain complex $C^c_*(X)$ is d-dimensional (resp. finite, resp. of finite type).*

Proposition 2.5 *Let $h : Z \longrightarrow Y$ be a G-map between G-CW-complexes such that both Z^H and Y^H are simply-connected for $H\in\mathcal{F}$ and all their isotropy groups belong to the family $\mathcal{F}$. Let $r\geq 2, r\geq\dim Z$ and a free $\mathbb{Z}\operatorname{Or}(G,\mathcal{F})$-chain complex (D_*,d_*) be given. Finally, suppose that there is a chain homotopy equivalence $f_* : D_* \longrightarrow C^c_*(Y,B)$ such that $D_*|_r = C^c_*(Z)|_r$ and $f_*|_r = C^c_*(h)|_r$.*

Then there is a G-CW-complex X with $X_r = Z$ and a cellular G-homotopy equivalence $g : X \longrightarrow Y$ such that:

i) $g|_Z = h$;

ii) $D_* = C^c_*(X)$;

iii) $C^c_*(g) = f_*$.

Proof: The proof is exactly the same as in [10, Theorem 13.19 on p. 268]. There only proper actions are considered but the same methods go through because here we are dealing with the easy case where all fixed point sets are simply connected, the isotropy groups belong to $\mathcal{F}$, and G is discrete. ■

Lemma 2.6 *If G is a discrete group and E is a G-CW-model for $E(G,\mathcal{F})$, then $C^c_*(E)$ is a free resolution over the orbit category* $\mathrm{Or}(G,\mathcal{F})$ *of the constant $Or(G,\mathcal{F})$-module $\underline{\mathbb{Z}}$ with value $\mathbb{Z}$.*

Proof: The modules are free by Lemma 2.2. It remains to show that $C^c_*(E^H)$, $H \in \mathcal{F}$ has the homology of a point. But this follows from the fact that for discrete H the space E^H has a canonical CW-structure whose n-skeleton in exactly E^H_n and E^H is weakly contractible and hence contractible. ■

To shorten the next proof we start with the following lemma. Its proof is purely technical and hence left out. Details of the proof can be found in [10, p. 279-280]. All modules are supposed to be over the orbit category.

Lemma 2.7 *Let C_* be a free, 2-dimensional chain complex, D_* a free chain complex and $f_* : C_* \longrightarrow D_*$ a chain map with $H_i(\mathrm{cone}(f_*)_*) = 0, i \leq 2$. Then there is a free chain complex C'_* and a chain homotopy equivalence $g_* : C'_* \longrightarrow D_*$ with $C'_*|_2 = C_*$ and $g_*|_2 = f_*$. If C_* is finite and D_* is homotopic to a finite free chain complex, resp. a free complex of finite type, then C'_* can be chosen to be finite, resp. of finite type. If D_* is homotopic to a finite-dimensional free complex, then C'_* can be chosen to be finite-dimensional.*

Proof of Theorem 0.1: The "only if"-case is clear for the first three assertions by Lemma 2.6 and Corollary 2.4. In the "if"-case for the first three assertions, let P_* be the given projective resolution of the constant $\mathbb{Z}Or(G,\mathcal{F})$-module $\underline{\mathbb{Z}}$ and let $E = E(G,\mathcal{F})$ be a G-CW-model with finite 2-skeleton in the second and third case. By adding elementary chain complexes, i.e. complexes concentrated in two consecutive dimensions with the identity as only non-trivial differential, we can get P_* to be a free resolution. (In the d-dimensional case we use the Eilenberg trick for the last module. Notice that in the finite case P_* is assumed to be free.) Since $C^c_*(E)$ also gives a free resolution of $\underline{\mathbb{Z}}$ by Lemma 2.6, we have a homotopy equivalence $g_* : P_* \longrightarrow C^c_*(E)$. Using Lemma 2.7, we get a new free complex Q_* with inherited finiteness property of P_* and a chain homotopy equivalence $f_* : Q_* \longrightarrow C^c_*(E)$ which induces the identity in dimensions 0,1 and 2. Therefore we can apply Proposition 2.5 to the inclusion $i : E_2 \hookrightarrow E$ and f_*. The result is a G-CW-complex X with $X_2 = E_2$ together with a **Comment:** `here we forgot the` G. G-homotopy equivalence $k : X \longrightarrow E$ and with $C^c_*(X) = Q_*$. So X is a G-CW-model for $E(G,\mathcal{F})$ and has the desired properties by Corollary 2.4.

The same proof as that of [11, Theorem 4.2], replacing the words "of finite type" by "with finite 2-skeleton", yields the last assertion of Theorem 0.1. ■

3 Totally Disconnected Groups

Recall that a topological space X is *totally disconnected* if any component consists of exactly one point. In this section we want to show that there is a close relation between **universal** spaces of a totally disconnected group G and those of its discretization G_d. The reason for this is that homotopy does not see the difference between a totally disconnected group G and G_d, i.e. the canonical map $G_d/H_d \longrightarrow \operatorname{res}_G^{G_d} G/H$ is a weak G_d-homotopy equivalence. The different topologies will only appear in the family of subgroups that has to be considered. We start by collecting some elementary facts about totally disconnected spaces.

Let X be a topological space. Consider the following 3 conditions.

(T) X is totally disconnected;

(D) The covering dimension of X is 0;

(FS) Any element of X has a fundamental system of open and compact neighborhoods.

Lemma 3.1 *For a locally compact group the conditions (T), (D) and (FS) are equivalent.*

Proof: The implications $(T) \Rightarrow (D) \Rightarrow (FS)$ are shown in [7, Theorem 7.7 on p. 62]. The implication $(FS) \Rightarrow (T)$ is done as follows: Let U be a set containing two distinct points x and y. We show that U is disconnected. Let V be an open and compact neighborhood of x, not containing y. Then $(V \cap U) \amalg (V^c \cap U) = U$ is a disjoint union of two nonempty open subsets of U. ■

The elementary proofs of the next two lemmas are left to the reader.

Lemma 3.2 *Let $f : X \longrightarrow Y$ be a surjective and open map . If X is locally compact, then Y is locally compact. If X has property (FS), then so does Y. In particular, if G is a totally disconnected locally compact group, then $(G/H)^K$ is totally disconnected and locally compact for all (closed) subgroups $H, K \subset G$.*

Lemma 3.3 *Let $f : X \longrightarrow Y$ be a map. If $f^{-1}(y)$ is weakly contractible for all $y \in Y$ and Y is totally disconnected, then f is a weak homotopy equivalence. If $f^{-1}(y)$ is contractible for all $y \in Y$ and Y is discrete, then f is a homotopy equivalence.*

Lemma 3.4 *Let G be a totally disconnected locally compact group and X be a G_d-CW-complex whose isotropy groups are all closed when viewed as subgroups of G. Then the map*

$$i_X : X \longrightarrow \operatorname{res}_G^{G_d} G \times_{G_d} X, \quad x \mapsto [e, x]$$

is a weak G_d-homotopy equivalence.

Proof: We begin with the case where X is a homogeneous space G_d/H_d for a closed subgroup $H \subset G$. Then the map

$$G \times_{G_d} G_d/H_d \longrightarrow G/H, \qquad [g, g'H_d] \mapsto gg'H$$

is a G-homeomorphism. The obvious map $G_d/H_d \to \operatorname{res}_G^{G_d} G/H$ is a weak G_d-homotopy equivalence by Lemma 3.2 and Lemma 3.3. Hence $i_{G_d/H_d} : G_d/H_d \longrightarrow \operatorname{res}_G^{G_d} G \times_{G_d} G_d/H_d$ is a weak G_d-homotopy equivalence.

Next we prove the claim for all skeleta X_n by induction over n. The **base case** $n \leq 0$ **of the induction** follows from the case of a homogeneous space since X_0 is a disjoint union of homogeneous spaces. In the induction step from $n-1$ to n one chooses a G_d-pushout

$$\begin{CD}
\coprod_{I_n} G_d/H_i \times S^{n-1} @>>> X_{n-1} \\
@VVV @VVV \\
\coprod_{I_n} G_d/H_i \times D^n @>>> X_n
\end{CD}$$

and checks, using the fact that G is locally compact, that the induced diagram is a G_d-pushout

$$\begin{CD}
\coprod_{I_n} \operatorname{res}_G^{G_d} G \times_{G_d} G_d/H_i \times S^{n-1} @>>> \operatorname{res}_G^{G_d} G \times_{G_d} X_{n-1} \\
@VVV @VVV \\
\coprod_{I_n} \operatorname{res}_G^{G_d} G \times_{G_d} G_d/H_i \times D^n @>>> \operatorname{res}_G^{G_d} G \times_{G_d} X_n.
\end{CD}$$

Notice that in both diagrams the left vertical arrows are G_d-cofibrations and the various maps i_Y for $Y = \coprod_{I_n} G_d/H_i \times S^{n-1}$, $Y = \coprod_{I_n} G_d/H_i \times D^n$, $Y = X_{n-1}$ and $Y = X_n$ map the two diagrams to one another. By the induction hypothesis the first three are weak G_d-homotopy equivalences. Hence i_{X_n} is a weak G_d-homotopy equivalence [10, Lemma 2.13 on p. 38].

Let $K \subset G_d$ be a subgroup. Since X^K has the weak topology with respect to the filtration given by the subspaces X_n^K and G is locally compact, $\left(\operatorname{res}_G^{G_d} G \times_{G_d} X\right)^K$ has the weak topology with respect to the filtration given by the subspaces $\left(\operatorname{res}_G^{G_d} G \times_{G_d} X_n\right)^K$. Since $(i_{X_n})^K$ is a weak homotopy equivalence for $n \geq 0$, the same follows for $(i_X)^K$. ■

Corollary 3.5 *Let G be a totally disconnected locally compact group and $\mathcal{F}$ a family of subgroups of G. If $E(G_d, \mathcal{F}_d)$ is a G_d-CW-model for the* **universal** *space of G_d for the family $\mathcal{F}_d$, then $G \times_{G_d} E(G_d, \mathcal{F}_d)$ is a G-CW-model for $E(G, \mathcal{F})$.*

Proof: We have for any $K \in \mathcal{F}$ by Lemma 3.4:

$$(G \times_{G_d} E(G_d, \mathcal{F}_d))^K = (\operatorname{res}_G^{G_d} G \times_{G_d} E(G_d, \mathcal{F}_d))^K \simeq_w E(G_d, \mathcal{F}_d)^{K_d} \simeq_w \{*\}. \qquad \blacksquare$$

Proposition 3.6 *Let G be totally disconnected and let X be a G-CW-complex that is d-dimensional (resp. finite, resp. of finite type). Then $\operatorname{res}_G^{G_d} X$ has a G_d-CW-approximation Y that is d-dimensional (resp. finite, resp. of finite type) and whose isotropy groups are the same as those of X. If X is a G-CW-model for $E(G, \mathcal{F})$, then Y is a G_d-CW-model for $E(G_d, \mathcal{F}_d)$.*

Proof: For $n \geq -1$ we construct by induction a G_d-CW-complex Y_n together with a G_d-approximation $f_n : Y_n \longrightarrow \operatorname{res}_G^{G_d} X_n$ such that Y_{n-1} is a subcomplex of Y_n and $f_n|_{Y_{n-1}} = f_{n-1}$. The **base case** $n = -1$ **of the induction** is given by the empty set. For the induction step from $n-1$ to n we proceed as follows. Choose a pushout

$$\begin{tikzcd}
\coprod_{I_n} G/H_i \times S^{n-1} \arrow[r, "r"] \arrow[d, hook] & X_{n-1} \arrow[d] \\
\coprod_{I_n} G/H_i \times D^n \arrow[r] & X_n
\end{tikzcd}$$

for the n-skeleton of X_n. Let $i : \coprod_{I_n} G_d/(H_i)_d \to \coprod_{I_n} \operatorname{res}_G^{G_d} G/H_i$ be the obvious weak G_d-homotopy equivalence (see Lemma 3.4). Since by the induction hypothesis f_{n-1} is a weak G_d-homotopy equivalence, we can find using [10, Proposition 2.3 on p. 35] and a version of the Cellular Approximation Theorem (see for instance [10, Theorem 2.1 on p. 32]) a cellular G_d-map $g : \coprod_{I_n} G_d/(H_i)_d \times S^{n-1} \longrightarrow Y_{n-1}$ and a G_d-homotopy $h : \coprod_{I_n} G_d/(H_i)_d \times S^{n-1} \times I \longrightarrow \operatorname{res}_G^{G_d} X_{n-1}$ between $f_{n-1} \circ g$ and $r \circ (i \times \operatorname{id}_{S^{n-1}})$. Let $f'_{n-1} : \operatorname{cyl}(g) \longrightarrow \operatorname{res}_G^{G_d} X_{n-1}$ be the obvious map given by h and f_{n-1}. Its restriction to $Y_{n-1} \subset \operatorname{cyl}(g)$ is f_{n-1} and to $\coprod_{I_n} G_d/(H_i)_d \times S^{n-1}$ is $r \circ (i \times \operatorname{id}_{S^{n-1}})$. Thus we obtain a commutative diagram

$$\begin{tikzcd}
\coprod_{I_n} G_d/(H_i)_d \times D^n \arrow[d, "i \times \mathrm{id}"] & \coprod_{I_n} G_d/(H_i)_d \times S^{n-1} \arrow[l, hook'] \arrow[r, hook] \arrow[d, "i \times \mathrm{id}"] & \operatorname{cyl}(g) \arrow[d, "f'_{n-1}"] \\
\coprod_{I_n} \operatorname{res}_G^{G_d} G/H_i \times D^n & \coprod_{I_n} \operatorname{res}_G^{G_d} G/H_i \times S^{n-1} \arrow[l, hook'] \arrow[r, "r"] & \operatorname{res}_G^{G_d} X_{n-1}.
\end{tikzcd}$$

Taking the pushout of the upper row yields a n-dimensional G_d-CW-complex Y_n which contains Y_{n-1} as G_d-CW-subcomplex and which is finite if X_n is finite. Moreover, we get by the pushout property a G_d-map $f_n : Y_n \to \operatorname{res}_G^{G_d} X_n$ which extends $f_{n-1} : Y_{n-1} \to \operatorname{res}_G^{G_d} X_{n-1}$ and is a weak G_d-homotopy equivalence, since all vertical maps are weak G_d-homotopy equivalences ([10, Lemma 2.13 on p. 38]). Now put $Y := \operatorname{colim}_{n\to\infty} Y_n$. Then $f := \operatorname{colim}_{n\to\infty} f_n : Y \to \operatorname{res}_G^{G_d} X$ is the G_d-CW-approximation we look for. ■

Proof of Theorem 0.2: Follows from Corollary 3.5 and Proposition 3.6. ■

4 Locally Compact Groups

The strategy of our study of locally compact groups is the following. Any locally compact group G gives rise to a short sequence of the form $1 \longrightarrow G_0 \longrightarrow G \xrightarrow{p} \overline{G} \longrightarrow 1$ with G_0 locally compact and connected and with $\overline{G}$ locally compact and totally disconnected. This reduces the study of G to the study of locally compact connected groups, which are very similar to Lie groups by the solution of Hilbert's fifth problem (cp. [6]), and to locally compact totally disconnected groups, which are similar to their discrete underlying group, as we saw in the preceding section. We start with some remarks on locally compact groups G which are *almost connected*, i.e. whose component group $\overline{G}$ is compact.

Theorem 4.1 *Let G be a almost connected locally compact group. Then G has a maximal compact subgroup K which is unique up to conjugacy and G/K is a model for both $\underline{E}G$ and $\underline{J}G := J(G, \mathcal{COM})$.*

Proof: [1, Appendix, Theorem A.5], [2, Corollary 4.14]. ■

We now turn our attention to locally compact groups that are not necessarily almost connected. From now on any locally compact group G is assumed to satisfy the condition (S) defined in the introduction.

Lemma 4.2 *Let L be an almost connected subgroup of G and let K be a maximal compact subgroup of L. If G/L is totally disconnected, then for any compact $H \subset G$ the projection $pr^H : (G/K)^H \longrightarrow (G/L)^H$ is a weak homotopy equivalence. If G/L is discrete, then for any compact $H \subset G$ the projection $pr^H : (G/K)^H \longrightarrow (G/L)^H$ is a homotopy equivalence.*

Proof: If H is not subconjugated to L, all spaces are empty. So, let H be subconjugated to L. Since we assumed the existence of local cross sections, we know that $G \longrightarrow G/L$ is a principal L-bundle. Let Y be any L-space. Fix an element $w \in G$. Then in the associated fiber bundle $Y \hookrightarrow G \times_L Y \xrightarrow{pr} G/L$, the typical fiber maps homeomorphically onto the preimage $p^{-1}(wL)$ of $wL \in G/L$ by sending y to the class of (w, y). If wL is in $(G/L)^H$ then this implies $wHw^{-1} \subset L$ and we get an induced homeomorphism $Y^{wHw^{-1}} \longrightarrow (pr^H)^{-1}(wL)$ for $pr^H : (G \times_L Y)^H \longrightarrow G/L^H$. Now let Y be L/K, which is a model for $J(L, \mathcal{COM})$ by Theorem 4.1. Therefore $Y^{wHw^{-1}}$ is contractible. Hence by Lemma 3.3 the map pr^H is a weak homotopy equivalence if G/L and hence $(G/L)^H$ is totally disconnected, and a homotopy equivalence, if G/L and hence $(G/L)^H$ is discrete. ■

Proposition 4.3 *Given a $\overline{G}$-CW-model $\underline{E}\overline{G}$, there is a G-CW-model $\underline{E}G$ and a G-map $f : \underline{E}G \longrightarrow p^*\underline{E}\overline{G}$ with the following properties (where $p^*\underline{E}\overline{G}$ is $\underline{E}\overline{G}$ viewed as a G-space by the projection $p : G \longrightarrow \overline{G}$):*

(a) If $\underline{E}\overline{G}$ is d-dimensional (resp. finite, resp. of finite type), then $\underline{E}G$ is d-dimensional (resp. finite, resp. of finite type);

(b) If $\overline{G}$ is discrete, then $\underline{E}G^H$ is contractible for all compact $H \subset G$;

(c) $G_0\backslash f : G_0\backslash\underline{E}G \longrightarrow \underline{E}\overline{G}$ is a $\overline{G}$-homotopy equivalence.

Proof: We will construct for each $n \geq -1$ an n-dimensional G-CW-complex X_n and a G-map $f_n : X_n \longrightarrow p^*\underline{E}\overline{G}_n$ to the n-skeleton of $p^*\underline{E}\overline{G}$ **Comment:** `Here we had the same statement as in (c).` with the following properties:

(a) $f_n^H : X_n^H \longrightarrow (p^*\underline{E}\overline{G}_n)^H$ is a weak homotopy equivalence for all compact $H \subset G$. If $\overline{G}$ is discrete, $f_n^H : X_n^H \longrightarrow (p^*\underline{E}\overline{G}_n)^H$ is a homotopy equivalence for all compact $H \subset G$;

(b) The isotropy groups of X_n are all compact;

(c) X_{n-1} is the $(n-1)$-skeleton of X_n and $f_n|_{X_{n-1}} = f_{n-1}$. There is a bijective correspondence between the equivariant n-dimensional cells of X_n and of $\underline{E}\overline{G}_n$;

(d) $G_0\backslash f_n : G_0\backslash X_n \longrightarrow \underline{E}\overline{G}_n$ is a $\overline{G}$-homotopy equivalence.

Notice that we then can define $\underline{E}G := \operatorname{colim}_{n\to\infty} X_n$ and $f := \operatorname{colim}_{n\to\infty} f_n$, and check that $\underline{E}G$ and f have the desired properties.

We proceed by induction over n. **The base case** $n = -1$ **of the induction** is given by $X_{-1} := \emptyset$. For the induction step from $n-1$ to n we choose a G-pushout

$$\begin{array}{ccc} \coprod_{I_n} \overline{G}/H_i \times S^{n-1} & \xrightarrow{\coprod_{I_n} q_i} & \underline{E}\overline{G}_{n-1} \\ \big\downarrow & & \big\downarrow \\ \coprod_{I_n} \overline{G}/H_i \times D^n & \longrightarrow & \underline{E}\overline{G}_n. \end{array}$$

Put $L_i := p^{-1}(H_i) \subset G$ for $i \in I_n$. Obviously L_i is almost connected. Let K_i be a maximal compact subgroup of L_i for $i \in I_n$ (see Theorem 4.1). Since the projection $p : G \longrightarrow \overline{G}$ induces a homeomorphism $G/L_i \xrightarrow{\cong} \overline{G}/H_i$, G/L_i is totally disconnected. Hence Lemma 4.2 implies that $pr_i^H : (G/K_i)^H \longrightarrow (G/L_i)^H$ is a weak homotopy equivalence for all compact subgroups H of G and is a homotopy equivalence for all compact subgroups H of G, provided that $\overline{G}$ is discrete. The same is true for f_{n-1} by the induction hypothesis. Therefore we have a bijection induced by f_{n-1} ([10, Prop. 2.3 on p.35])

$$[G/K_i \times S^{n-1}, X_{n-1}]_G \xrightarrow{(f_{n-1})_*} [G/K_i \times S^{n-1}, p^*\underline{E}\overline{G}_{n-1}]_G.$$

Using the Equivariant Cellular Approximation Theorem [10, Theorem 2.1 on p. 32] we get a cellular G-map $r_i : G/K_i \times S^{n-1} \to X_{n-1}$ together with a G-homotopy $h_i : G/K_i \times S^{n-1} \times [0,1] \to p^*\underline{E}\overline{G}_{n-1}$ from $f_{n-1} \circ r_i$ to $q_i \circ (pr_i \times \mathrm{id}_{S^{n-1}})$. Consider the following commutative G-diagram

$$\begin{tikzcd}
\coprod_{I_n} p^*(\overline{G}/H_i) \times D^n & \coprod_{I_n} p^*(\overline{G}/H_i) \times S^{n-1} \arrow[l, hook'] \arrow[r, "\coprod_{I_n} q_i"] & p^*\underline{E}\overline{G}_{n-1} \\
\coprod_{I_n} G/K_i \times D^n \arrow[u, "\coprod_{I_n} pr_i \times \mathrm{id}_{D^n}"] \arrow[d, "i_1"'] & \coprod_{I_n} G/K_i \times S^{n-1} \arrow[l, hook'] \arrow[r, "\coprod_{I_n} q_i \circ (pr_i \times \mathrm{id}_{S^{n-1}})"] \arrow[u, "\coprod_{I_n} pr_i \times \mathrm{id}_{S^{n-1}}"] \arrow[d, "i_1"'] & p^*\underline{E}\overline{G}_{n-1} \arrow[u, "\mathrm{id}"'] \arrow[d, "\mathrm{id}"'] \\
\coprod_{I_n} G/K_i \times D^n \times [0,1] & \coprod_{I_n} G/K_i \times S^{n-1} \times [0,1] \arrow[l, hook'] \arrow[r, "\coprod_{I_n} h_i"] & p^*\underline{E}\overline{G}_{n-1} \\
\coprod_{I_n} G/K_i \times D^n \arrow[u, "i_0"] & \coprod_{I_n} G/K_i \times S^{n-1} \arrow[l, hook'] \arrow[r, "\coprod_{I_n} r_i"] \arrow[u, "i_0"] & X_{n-1}. \arrow[u, "f_{n-1}"']
\end{tikzcd}$$

Notice that the pushout of the first row is $p^*\underline{E}\overline{G}_n$. Denote the pushout of the second, third and fourth row respectively by X_n', X_n'' and X_n. The diagram above together with the pushout property induces G-maps $f_n' : X_n' \longrightarrow p^*\underline{E}\overline{G}_n$, $f_n'' : X_n' \longrightarrow X_n''$ and $f_n''' : X_n \to X_n''$. The map f_n'' is a G-homotopy equivalence and the maps $(f_n')^H$ and$(f_n''')^H$ are weak homotopy equivalences (homotopy equivalence if $\overline{G}$ is discrete) for each compact subgroup $H \subset G$ ([10, Lemma 2.13 on p. 38]). We can choose a G-map $(f_n'')^{-1} : X_n'' \to X_n'$ which induces the identity on $p^*\underline{E}\overline{G}_{n-1}$ and is a G-homotopy inverse of f_n''. Now define $f_n : X_n \longrightarrow p^*\underline{E}\overline{G}_n$ by the composition $f_n' \circ (f_n'')^{-1} \circ f_n'''$. By construction f_n^H is a weak homotopy equivalence (homotopy equivalence if $\overline{G}$ is discrete) for all compact subgroups $H \subset G$ and X_n is a G-CW-complex with X_{n-1} as its $(n-1)$-skeleton and has only compact isotropy groups.

It remains to show that $G_0 \backslash f_n : G_0 \backslash X_n \to \underline{E}\overline{G}_n$ is a $\overline{G}$-homotopy equivalence. Since L_i inherits the property (S) from G, we get a locally trivial fiber bundle $K_i \longrightarrow L_i \longrightarrow L_i/K_i$ which is automatically a Serre fibration and hence induces a long exact homotopy sequence [14, Theorem 2.11 on p. 60, Theorem 3.6 on p. 65 and Corollary 3.11 on p. 67]. Thus we get the following diagram

$$\begin{array}{ccccccc} & & K_i & & & & \\ & & \downarrow & & & & \\ 1 = \pi_1(L_i/K_i) & \longrightarrow & \pi_0(K_i) & \xrightarrow{\cong} & \pi_0(L_i) & \longrightarrow & \pi_0(L_i/K_i) = 1. \\ & & & & \downarrow & & \\ & & & & G_0\backslash L_i = H_i & & \end{array}$$

We conclude that $p(K_i) = H_i$ holds for all $i \in I_n$. Hence $G_0\backslash pr_i : G_0\backslash G/K_i \longrightarrow \overline{G}/H_i$ is a $\overline{G}$-homeomorphism. Therefore $G_0\backslash f_n'$ is a $\overline{G}$-homeomorphism. $G_0\backslash(f_n'')^{-1}$ is a $\overline{G}$-homotopy equivalence, since $(f_n'')^{-1}$ is a G-homotopy equivalence, and $G_0\backslash(f_n''')$ is a $\overline{G}$-homotopy equivalence, since $G_0\backslash f_{n-1}$ is a $\overline{G}$-homotopy equivalence by the induction hypothesis. Hence $G_0\backslash f_n$ is a $\overline{G}$-homotopy equivalence. ∎

Proof of Theorem 0.3: Is implied by Proposition 4.3. ∎

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