Equivariant homology theories (Lecture IV)

Wolfgang Lück
Münster
Germany
email lueck@math.uni-muenster.de
http://www.math.uni-muenster.de/u/lueck/

Hangzhou, July 2007

Flashback

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
  Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group G which we will recall next.
- In the sequel group will mean discrete group.

Flashback

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
  Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group G which we will recall next.
- In the sequel group will mean discrete group.

Flashback

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
  Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group G which we will recall next.
- In the sequel group will mean discrete group.

Flashback

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
  Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group G which we will recall next.
- In the sequel group will mean discrete group.

Flashback

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
  Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group G which we will recall next.
- In the sequel group will mean discrete group.

Flashback

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
  Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group G which we will recall next.
- In the sequel group will mean discrete group.

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$\begin{aligned}
\mathcal{TR} &= \{\text{trivial subgroup}\};\\
\mathcal{FIN} &= \{\text{finite subgroups}\};\\
\mathcal{FCYC} &= \{\text{finite cyclic subgroups}\};\\
\mathcal{VCYC} &= \{\text{virtually cyclic subgroups}\};\\
\mathcal{ALL} &= \{\text{all subgroups}\}.
\end{aligned}$$

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$\begin{array}{lcl} \mathcal{TR} & = & \{\text{trivial subgroup}\}; \\ \mathcal{FIN} & = & \{\text{finite subgroups}\}; \\ \mathcal{FCYC} & = & \{\text{finite cyclic subgroups}\}; \\ \mathcal{VCYC} & = & \{\text{virtually cyclic subgroups}\}; \\ \mathcal{ALL} & = & \{\text{all subgroups}\}. \end{array}$$

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$
\begin{array}{lcl}
\mathcal{TR} & = & \{\text{trivial subgroup}\}; \\
\mathcal{FIN} & = & \{\text{finite subgroups}\}; \\
\mathcal{FCYC} & = & \{\text{finite cyclic subgroups}\}; \\
\mathcal{VCYC} & = & \{\text{virtually cyclic subgroups}\}; \\
\mathcal{ALL} & = & \{\text{all subgroups}\}.
\end{array}
$$

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$\begin{array}{lcl} \mathcal{TR} & = & \{\text{trivial subgroup}\}; \\ \mathcal{FIN} & = & \{\text{finite subgroups}\}; \\ \mathcal{FCYC} & = & \{\text{finite cyclic subgroups}\}; \\ \mathcal{VCYC} & = & \{\text{virtually cyclic subgroups}\}; \\ \mathcal{ALL} & = & \{\text{all subgroups}\}. \end{array}$$

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$\begin{array}{lcl} \mathcal{TR} & = & \{\text{trivial subgroup}\}; \\ \mathcal{FIN} & = & \{\text{finite subgroups}\}; \\ \mathcal{FCYC} & = & \{\text{finite cyclic subgroups}\}; \\ \mathcal{VCYC} & = & \{\text{virtually cyclic subgroups}\}; \\ \mathcal{ALL} & = & \{\text{all subgroups}\}. \end{array}$$

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$\begin{array}{lcl} \mathcal{TR} & = & \{\text{trivial subgroup}\}; \\ \mathcal{FIN} & = & \{\text{finite subgroups}\}; \\ \mathcal{FCYC} & = & \{\text{finite cyclic subgroups}\}; \\ \mathcal{VCYC} & = & \{\text{virtually cyclic subgroups}\}; \\ \mathcal{ALL} & = & \{\text{all subgroups}\}. \end{array}$$

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$\mathcal{TR}$	=	{trivial subgroup};
$\mathcal{FIN}$	=	{finite subgroups};
$\mathcal{FCYC}$	=	{finite cyclic subgroups};
$\mathcal{VCYC}$	=	{virtually cyclic subgroups};
$\mathcal{ALL}$	=	{all subgroups}.

Definition (Family of subgroups)

A *family $\mathcal{F}$ of subgroups* of G is a set of subgroups of G which is closed under conjugation and finite intersections.

Examples for $\mathcal{F}$ are:

$$
\begin{array}{lcl}
\mathcal{TR} & = & \{\text{trivial subgroup}\}; \\
\mathcal{FIN} & = & \{\text{finite subgroups}\}; \\
\mathcal{FCYC} & = & \{\text{finite cyclic subgroups}\}; \\
\mathcal{VCYC} & = & \{\text{virtually cyclic subgroups}\}; \\
\mathcal{ALL} & = & \{\text{all subgroups}\}.
\end{array}
$$

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.
We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.

We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.
We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.

We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.
We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.

We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.
We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

Definition (Classifying G-CW-complex for a family of subgroups)

Let $\mathcal{F}$ be a family of subgroups of G. A model for the *classifying G-CW-complex for the family $\mathcal{F}$* is a G-CW-complex $E_{\mathcal{F}}(G)$ which has the following properties:

- All isotropy groups of $E_{\mathcal{F}}(G)$ belong to $\mathcal{F}$;
- For any G-CW-complex Y, whose isotropy groups belong to $\mathcal{F}$, there is up to G-homotopy precisely one G-map $Y \to X$.

We abbreviate $\underline{E}G := E_{\mathcal{FIN}}(G)$ and call it the *universal G-CW-complex for proper G-actions*.
We also write $EG = E_{\mathcal{TR}}(G)$.

- A model for $E_{\mathcal{F}}(G)$ exists and is unique up to G-homotopy.

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

- Cliffhanger

Question (Homological computations based on nice models for $\underline{E}G$)

Can nice geometric models for $\underline{E}G$ be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (K-theory of group rings and group homology)

Is there a relation between $K_n(RG)$ and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

Outline

- We intoduce the notion of an equivariant homology theory.
- We present the general formulation of the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We discuss equivariant Chern characters.
- We present some explicit computations of equivariant topological K-groups and of homology groups associated to classifying spaces of groups.

Outline

- We intoduce the notion of an equivariant homology theory.
- We present the general formulation of the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We discuss equivariant Chern characters.
- We present some explicit computations of equivariant topological K-groups and of homology groups associated to classifying spaces of groups.

Outline

- We intoduce the notion of an equivariant homology theory.
- We present the general formulation of the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We discuss equivariant Chern characters.
- We present some explicit computations of equivariant topological K-groups and of homology groups associated to classifying spaces of groups.

Outline

- We intoduce the notion of an equivariant homology theory.
- We present the general formulation of the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We discuss equivariant Chern characters.
- We present some explicit computations of equivariant topological K-groups and of homology groups associated to classifying spaces of groups.

Outline

- We intoduce the notion of an equivariant homology theory.
- We present the general formulation of the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We discuss equivariant Chern characters.
- We present some explicit computations of equivariant topological K-groups and of homology groups associated to classifying spaces of groups.

Equivariant homology theories

Definition (G-homology theory)

A *G-homology theory $\mathcal{H}_*$* is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X, A) \colon \mathcal{H}_n(X, A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Equivariant homology theories

Definition (G-homology theory)

A G-*homology theory* $\mathcal{H}_*$ is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X, A) \colon \mathcal{H}_n(X, A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Equivariant homology theories

Definition (G-homology theory)

A *G-homology theory $\mathcal{H}_*$* is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X, A) \colon \mathcal{H}_n(X, A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Equivariant homology theories

Definition (G-homology theory)

A *G-homology theory* $\mathcal{H}_*$ is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X,A)\colon \mathcal{H}_n(X,A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Equivariant homology theories

Definition (G-homology theory)

A *G-homology theory* $\mathcal{H}_*$ is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X, A) \colon \mathcal{H}_n(X, A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Equivariant homology theories

Definition (G-homology theory)

A *G-homology theory* $\mathcal{H}_*$ is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X, A)\colon \mathcal{H}_n(X, A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Equivariant homology theories

Definition (G-homology theory)

A *G-homology theory* $\mathcal{H}_*$ is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n(X, A) \colon \mathcal{H}_n(X, A) \to \mathcal{H}_{n-1}(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Definition (*G*-homology theory)

A *G-homology theory* $\mathcal{H}_*^G$ is a covariant functor from the category of G-CW-pairs to the category of $\mathbb{Z}$-graded Λ-modules together with natural transformations

$$\partial_n^G(X,A)\colon \mathcal{H}_n^G(X,A) \to \mathcal{H}_{n-1}^G(A)$$

for $n \in \mathbb{Z}$ satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
- Excision;
- Disjoint union axiom.

Definition (Equivalent homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha\colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\mathrm{ind}_\alpha \colon \mathcal{H}_n^H(X, A) \quad \to \quad \mathcal{H}_n^G(\mathrm{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then ind_α is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivariant homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha\colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\mathrm{ind}_\alpha\colon \mathcal{H}_n^H(X, A) \rightarrow \mathcal{H}_n^G(\mathrm{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then ind_α is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivariant homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha \colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\mathrm{ind}_\alpha \colon \mathcal{H}_n^H(X, A) \quad \to \quad \mathcal{H}_n^G(\mathrm{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then ind_α is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivariant homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha\colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\operatorname{ind}_\alpha\colon \mathcal{H}_n^H(X, A) \quad \to \quad \mathcal{H}_n^G(\operatorname{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then $\operatorname{ind}_\alpha$ is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivalent homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha\colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\operatorname{ind}_\alpha \colon \mathcal{H}_n^H(X, A) \quad\to\quad \mathcal{H}_n^G(\operatorname{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then $\operatorname{ind}_\alpha$ is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivariant homology theory)

An *equivariant homology theory* $\mathcal{H}^?_*$ assigns to every group G a G-homology theory $\mathcal{H}^G_*$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha \colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\operatorname{ind}_\alpha \colon \mathcal{H}^H_n(X, A) \quad \to \quad \mathcal{H}^G_n(\operatorname{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then $\operatorname{ind}_\alpha$ is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivariant homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha\colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\mathrm{ind}_\alpha\colon \mathcal{H}_n^H(X, A) \quad\to\quad \mathcal{H}_n^G(\mathrm{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then ind_α is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Definition (Equivariant homology theory)

An *equivariant homology theory* $\mathcal{H}_*^?$ assigns to every group G a G-homology theory $\mathcal{H}_*^G$. These are linked together with the following so called *induction structure*: given a group homomorphism $\alpha\colon H \to G$ and a H-CW-pair (X, A), there are for all $n \in \mathbb{Z}$ natural homomorphisms

$$\mathrm{ind}_\alpha\colon \mathcal{H}_n^H(X, A) \;\to\; \mathcal{H}_n^G(\mathrm{ind}_\alpha(X, A))$$

satisfying

- Bijectivity
  If $\ker(\alpha)$ acts freely on X, then ind_α is a bijection;
- Compatibility with the boundary homomorphisms;
- Functoriality in α;
- Compatibility with conjugation.

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned}\mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G);\\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}.\end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E}\colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned} \mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G); \\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology).} \end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E}\colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\mathcal{H}_*^G(X) := \mathcal{K}_*(X/G);$$

$$\mathcal{H}_*^G(X) := \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology).}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E}\colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned} \mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G); \\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}. \end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E}\colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned} \mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G); \\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}. \end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E} \colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned} \mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G); \\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}. \end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E}\colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned} \mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G); \\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}. \end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E}\colon \mathrm{Groupoids} \to \mathrm{Spectra}$ sending equivalences to weak equivalences, there exists an equivariant homology theory $\mathcal{H}_*^?(-;\mathbf{E})$ satisfying

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned}\mathcal{H}^G_*(X) &:= \mathcal{K}_*(X/G);\\ \mathcal{H}^G_*(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}.\end{aligned}$$

- Equivariant bordism $\Omega^?_*(X)$;
- Equivariant topological K-theory $K^?_*(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E} \colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}^?_*(-;\mathbf{E})$ *satisfying*

$$\mathcal{H}^H_n(pt) \cong \mathcal{H}^G_n(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Example (Equivariant homology theories)

- Given a non-equivariant homology theory $\mathcal{K}_*$, put

$$\begin{aligned} \mathcal{H}_*^G(X) &:= \mathcal{K}_*(X/G); \\ \mathcal{H}_*^G(X) &:= \mathcal{K}_*(EG \times_G X) \quad \text{(Borel homology)}. \end{aligned}$$

- Equivariant bordism $\Omega_*^?(X)$;
- Equivariant topological K-theory $K_*^?(X)$.

Theorem (L.-Reich (2005))

Given a functor $\mathbf{E} \colon \mathrm{Groupoids} \to \mathrm{Spectra}$ *sending equivalences to weak equivalences, there exists an equivariant homology theory* $\mathcal{H}_*^?(-; \mathbf{E})$ *satisfying*

$$\mathcal{H}_n^H(pt) \cong \mathcal{H}_n^G(G/H) \cong \pi_n(\mathbf{E}(H)).$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned} \mathbf{K}_R \colon \text{Groupoids} &\to \text{Spectra}; \\ \mathbf{L}_R^{\langle \infty \rangle} \colon \text{Groupoids} &\to \text{Spectra}; \\ \mathbf{K}^{\text{top}} \colon \text{Groupoids}^{\text{inj}} &\to \text{Spectra} \end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned} \pi_n(\mathbf{K}_R(G)) &\cong K_n(RG); \\ \pi_n(\mathbf{L}_R^{\langle -\infty \rangle}(G)) &\cong L_n^{\langle -\infty \rangle}(RG); \\ \pi_n(\mathbf{K}^{\text{top}}(G)) &\cong K_n(C_r^*(G)). \end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned}\mathbf{K}_R \colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\ \mathbf{L}_R^{\langle\infty\rangle} \colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\ \mathbf{K}^{\mathrm{top}} \colon \mathrm{Groupoids}^{\mathrm{inj}} &\to \mathrm{Spectra}\end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned}\pi_n(\mathbf{K}_R(G)) &\cong K_n(RG);\\ \pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) &\cong L_n^{\langle-\infty\rangle}(RG);\\ \pi_n(\mathbf{K}^{\mathrm{top}}(G)) &\cong K_n(C_r^*(G)).\end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned} \mathbf{K}_R \colon \text{Groupoids} &\to \text{Spectra}; \\ \mathbf{L}_R^{\langle\infty\rangle} \colon \text{Groupoids} &\to \text{Spectra}; \\ \mathbf{K}^{\text{top}} \colon \text{Groupoids}^{\text{inj}} &\to \text{Spectra} \end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned} \pi_n(\mathbf{K}_R(G)) &\cong K_n(RG); \\ \pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) &\cong L_n^{\langle-\infty\rangle}(RG); \\ \pi_n(\mathbf{K}^{\text{top}}(G)) &\cong K_n(C_r^*(G)). \end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned}
\mathbf{K}_R \colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\
\mathbf{L}_R^{\langle\infty\rangle} \colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\
\mathbf{K}^{\mathrm{top}} \colon \mathrm{Groupoids}^{\mathrm{inj}} &\to \mathrm{Spectra}
\end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned}
\pi_n(\mathbf{K}_R(G)) &\cong K_n(RG);\\
\pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) &\cong L_n^{\langle-\infty\rangle}(RG);\\
\pi_n(\mathbf{K}^{\mathrm{top}}(G)) &\cong K_n(C_r^*(G)).
\end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned} \mathbf{K}_R \colon \mathrm{Groupoids} &\to \mathrm{Spectra}; \\ \mathbf{L}_R^{\langle \infty \rangle} \colon \mathrm{Groupoids} &\to \mathrm{Spectra}; \\ \mathbf{K}^{\mathrm{top}} \colon \mathrm{Groupoids}^{\mathrm{inj}} &\to \mathrm{Spectra} \end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned} \pi_n(\mathbf{K}_R(G)) &\cong K_n(RG); \\ \pi_n(\mathbf{L}_R^{\langle -\infty \rangle}(G)) &\cong L_n^{\langle -\infty \rangle}(RG); \\ \pi_n(\mathbf{K}^{\mathrm{top}}(G)) &\cong K_n(C_r^*(G)). \end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned}
\mathbf{K}_R \colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\
\mathbf{L}_R^{\langle\infty\rangle} \colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\
\mathbf{K}^{\mathrm{top}} \colon \mathrm{Groupoids}^{\mathrm{inj}} &\to \mathrm{Spectra}
\end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned}
\pi_n(\mathbf{K}_R(G)) &\cong K_n(RG);\\
\pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) &\cong L_n^{\langle-\infty\rangle}(RG);\\
\pi_n(\mathbf{K}^{\mathrm{top}}(G)) &\cong K_n(C_r^*(G)).
\end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned} \mathbf{K}_R \colon \mathsf{Groupoids} &\to \mathsf{Spectra}; \\ \mathbf{L}_R^{\langle\infty\rangle} \colon \mathsf{Groupoids} &\to \mathsf{Spectra}; \\ \mathbf{K}^{\mathrm{top}} \colon \mathsf{Groupoids}^{\mathrm{inj}} &\to \mathsf{Spectra} \end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned} \pi_n(\mathbf{K}_R(G)) &\cong K_n(RG); \\ \pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) &\cong L_n^{\langle-\infty\rangle}(RG); \\ \pi_n(\mathbf{K}^{\mathrm{top}}(G)) &\cong K_n(C_r^*(G)). \end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned} \mathbf{K}_R \colon \text{Groupoids} &\to \text{Spectra}; \\ \mathbf{L}_R^{\langle\infty\rangle} \colon \text{Groupoids} &\to \text{Spectra}; \\ \mathbf{K}^{\text{top}} \colon \text{Groupoids}^{\text{inj}} &\to \text{Spectra} \end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned} \pi_n(\mathbf{K}_R(G)) &\cong K_n(RG); \\ \pi_n(\mathbf{L}_R^{\langle -\infty\rangle}(G)) &\cong L_n^{\langle -\infty\rangle}(RG); \\ \pi_n(\mathbf{K}^{\text{top}}(G)) &\cong K_n(C_r^*(G)). \end{aligned}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{array}{rcl} \mathbf{K}_R \colon \text{Groupoids} & \to & \text{Spectra}; \\ \mathbf{L}_R^{\langle\infty\rangle} \colon \text{Groupoids} & \to & \text{Spectra}; \\ \mathbf{K}^{\text{top}} \colon \text{Groupoids}^{\text{inj}} & \to & \text{Spectra} \end{array}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{array}{rcl} \pi_n(\mathbf{K}_R(G)) & \cong & K_n(RG); \\ \pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) & \cong & L_n^{\langle-\infty\rangle}(RG); \\ \pi_n(\mathbf{K}^{\text{top}}(G)) & \cong & K_n(C_r^*(G)). \end{array}$$

Theorem (Equivariant homology theories associated to K and L-theory, Davis-L. (1998))

Let R be a ring (with involution). There exist covariant functors

$$\begin{aligned}\mathbf{K}_R\colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\ \mathbf{L}_R^{\langle\infty\rangle}\colon \mathrm{Groupoids} &\to \mathrm{Spectra};\\ \mathbf{K}^{\mathrm{top}}\colon \mathrm{Groupoids}^{\mathrm{inj}} &\to \mathrm{Spectra}\end{aligned}$$

with the following properties:

- *They send equivalences of groupoids to weak equivalences of spectra;*
- *For every group G and all $n \in \mathbb{Z}$ we have*

$$\begin{aligned}\pi_n(\mathbf{K}_R(G)) &\cong K_n(RG);\\ \pi_n(\mathbf{L}_R^{\langle-\infty\rangle}(G)) &\cong L_n^{\langle-\infty\rangle}(RG);\\ \pi_n(\mathbf{K}^{\mathrm{top}}(G)) &\cong K_n(C_r^*(G)).\end{aligned}$$

Example (Equivariant homology theories associated to K and L-theory)

We get equivariant homology theories

$$H_*^?(-;\mathbf{K}_R);$$
$$H_*^?(-;\mathbf{L}_R^{\langle -\infty\rangle});$$
$$H_*^?(-;\mathbf{K}^{\mathrm{top}}),$$

satisfying for $H \subseteq G$

$$\begin{array}{lclcl} H_n^G(G/H;\mathbf{K}_R) & \cong & H_n^H(\mathrm{pt};\mathbf{K}_R) & \cong & K_n(RH); \\ H_n^G(G/H;\mathbf{L}_R^{\langle -\infty\rangle}) & \cong & H_n^H(\mathrm{pt};\mathbf{L}_R^{\langle -\infty\rangle}) & \cong & L_n^{\langle -\infty\rangle}(RH); \\ H_n^G(G/H;\mathbf{K}^{\mathrm{top}}) & \cong & H_n^H(\mathrm{pt};\mathbf{K}^{\mathrm{top}}) & \cong & K_n(C_r^*(H)). \end{array}$$

Example (Equivariant homology theories associated to K and L-theory)

We get equivariant homology theories

$$H_*^?(-;\mathbf{K}_R);$$
$$H_*^?(-;\mathbf{L}_R^{\langle -\infty\rangle});$$
$$H_*^?(-;\mathbf{K}^{\mathrm{top}}),$$

satisfying for $H \subseteq G$

$$\begin{array}{lclcl} H_n^G(G/H;\mathbf{K}_R) & \cong & H_n^H(\mathrm{pt};\mathbf{K}_R) & \cong & K_n(RH); \\ H_n^G(G/H;\mathbf{L}_R^{\langle -\infty\rangle}) & \cong & H_n^H(\mathrm{pt};\mathbf{L}_R^{\langle -\infty\rangle}) & \cong & L_n^{\langle -\infty\rangle}(RH); \\ H_n^G(G/H;\mathbf{K}^{\mathrm{top}}) & \cong & H_n^H(\mathrm{pt};\mathbf{K}^{\mathrm{top}}) & \cong & K_n(C_r^*(H)). \end{array}$$

Example (Equivariant homology theories associated to K and L-theory)

We get equivariant homology theories

$$H^?_*(-;\mathbf{K}_R);$$
$$H^?_*(-;\mathbf{L}_R^{\langle -\infty\rangle});$$
$$H^?_*(-;\mathbf{K}^{\mathrm{top}}),$$

satisfying for $H \subseteq G$

$$\begin{array}{lclcl}
H_n^G(G/H;\mathbf{K}_R) & \cong & H_n^H(\mathrm{pt};\mathbf{K}_R) & \cong & K_n(RH); \\
H_n^G(G/H;\mathbf{L}_R^{\langle -\infty\rangle}) & \cong & H_n^H(\mathrm{pt};\mathbf{L}_R^{\langle -\infty\rangle}) & \cong & L_n^{\langle -\infty\rangle}(RH); \\
H_n^G(G/H;\mathbf{K}^{\mathrm{top}}) & \cong & H_n^H(\mathrm{pt};\mathbf{K}^{\mathrm{top}}) & \cong & K_n(C_r^*(H)).
\end{array}$$

The general formulation of the Isomorphism Conjectures

Conjecture (*K*-theoretic Farrell-Jones-Conjecture)

The K-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{K}_R) \to H_n^G(pt, \mathbf{K}_R) = K_n(RG)$$

is bijective for all $n \in \mathbb{Z}$.

- The assembly map is the map induced by the projection $E_{\mathcal{VCYC}}(G) \to \mathrm{pt}$.

The general formulation of the Isomorphism Conjectures

Conjecture (*K*-theoretic Farrell-Jones-Conjecture)

The K-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{K}_R) \to H_n^G(pt, \mathbf{K}_R) = K_n(RG)$$

is bijective for all $n \in \mathbb{Z}$.

- The assembly map is the map induced by the projection $E_{\mathcal{VCYC}}(G) \to \text{pt}$.

The general formulation of the Isomorphism Conjectures

Conjecture (*K*-theoretic Farrell-Jones-Conjecture)

The K-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{K}_R) \to H_n^G(pt, \mathbf{K}_R) = K_n(RG)$$

is bijective for all $n \in \mathbb{Z}$.

- The assembly map is the map induced by the projection $E_{\mathcal{VCYC}}(G) \to \mathrm{pt}$.

The general formulation of the Isomorphism Conjectures

Conjecture (*K*-theoretic Farrell-Jones-Conjecture)

The K-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{K}_R) \to H_n^G(pt, \mathbf{K}_R) = K_n(RG)$$

is bijective for all $n \in \mathbb{Z}$.

- The assembly map is the map induced by the projection $E_{\mathcal{VCYC}}(G) \to \mathrm{pt}$.

Conjecture (*L*-theoretic Farrell-Jones-Conjecture)

The L-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{L}_R^{\langle -\infty \rangle}) \to H_n^G(pt, \mathbf{L}_R^{\langle -\infty \rangle}) = L_n^{\langle -\infty \rangle}(RG)$$

is bijective for all $n \in \mathbb{Z}$.

Conjecture (*L*-theoretic Farrell-Jones-Conjecture)

The L-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{L}_R^{\langle -\infty \rangle}) \to H_n^G(pt, \mathbf{L}_R^{\langle -\infty \rangle}) = L_n^{\langle -\infty \rangle}(RG)$$

is bijective for all $n \in \mathbb{Z}$.

Conjecture (Baum-Connes Conjecture)

The Baum-Connes Conjecture predicts that the assembly map

$$K_n^G(\underline{E}G) = H_n^G(E_{\mathcal{FIN}}(G), \mathbf{K}^{\mathrm{top}}) \to H_n^G(pt, \mathbf{K}^{\mathrm{top}}) = K_n(C_r^*(G))$$

is bijective for all $n \in \mathbb{Z}$.

- We will discuss these conjectures and their applications in the next lecture.
- We will now continue with equivariant homology theories.

Conjecture (Baum-Connes Conjecture)

The Baum-Connes Conjecture predicts that the assembly map

$$K_n^G(\underline{E}G) = H_n^G(E_{\mathcal{FIN}}(G), \mathbf{K}^{\mathrm{top}}) \to H_n^G(pt, \mathbf{K}^{\mathrm{top}}) = K_n(C_r^*(G))$$

is bijective for all $n \in \mathbb{Z}$.

- We will discuss these conjectures and their applications in the next lecture.
- We will now continue with equivariant homology theories.

Conjecture (Baum-Connes Conjecture)

The Baum-Connes Conjecture predicts that the assembly map

$$K_n^G(\underline{E}G) = H_n^G(E_{\mathcal{FIN}}(G), \mathbf{K}^{\mathrm{top}}) \to H_n^G(pt, \mathbf{K}^{\mathrm{top}}) = K_n(C_r^*(G))$$

is bijective for all $n \in \mathbb{Z}$.

- We will discuss these conjectures and their applications in the next lecture.
- We will now continue with equivariant homology theories.

Conjecture (Baum-Connes Conjecture)

The Baum-Connes Conjecture predicts that the assembly map

$$K_n^G(\underline{E}G) = H_n^G(E_{\mathcal{FIN}}(G), \mathbf{K}^{\mathrm{top}}) \to H_n^G(pt, \mathbf{K}^{\mathrm{top}}) = K_n(C_r^*(G))$$

is bijective for all $n \in \mathbb{Z}$.

- We will discuss these conjectures and their applications in the next lecture.
- We will now continue with equivariant homology theories.

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X; \Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X; \Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X; \Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X; \Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X; \Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X;\Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Equivariant Chern characters

- Let $\mathcal{H}_*$ be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to $\mathcal{H}_{p+q}(X)$ and has as E^2-term

$$E^2_{p,q} = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

- Rationally it collapses completely. Namely, one has the following result

Theorem (Non-equivariant Chern character, Dold (1962))

Let $\mathcal{H}_*$ be a homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Then there exists for every $n \in \mathbb{Z}$ and every CW-complex X a natural isomorphism

$$\bigoplus_{p+q=n} H_p(X; \Lambda) \otimes_\Lambda \mathcal{H}_q(pt) \xrightarrow{\cong} \mathcal{H}_n(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X; \mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X; \mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n} (\mathrm{hur} \otimes \mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+, *) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f \colon (S^{p+k}, \mathrm{pt}) \to (S^k \wedge X_+, \mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k}, \mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+, \mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes\mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n\colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes\mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes\mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n\colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes\mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes \mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n\colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes\mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes \mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes \mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

Dold's Chern character for a CW-complex X is given by the following composite:

$$\mathrm{ch}_n \colon \bigoplus_{p+q=n} H_p(X;\mathcal{H}_q(*)) \xrightarrow{\alpha^{-1}} \bigoplus_{p+q=n} H_p(X;\mathbb{Z}) \otimes_{\mathbb{Z}} \mathcal{H}_q(*)$$

$$\xrightarrow{\bigoplus_{p+q=n}(\mathrm{hur}\otimes\mathrm{id})^{-1}} \bigoplus_{p+q=n} \pi_p^s(X_+,*) \otimes_{\mathbb{Z}} \mathcal{H}_q(*) \xrightarrow{\bigoplus_{p+q=n} D_{p,q}} \mathcal{H}_n(X),$$

where $D_{p,q}$ sends $[f\colon (S^{p+k},\mathrm{pt}) \to (S^k \wedge X_+,\mathrm{pt})] \otimes \eta$ to the image of η under the composite

$$\mathcal{H}_q(*) \cong \mathcal{H}_{p+k+q}(S^{p+k},\mathrm{pt}) \xrightarrow{\mathcal{H}_{p+k+q}(f)} \mathcal{H}_{p+k+q}(S^k \wedge X_+,\mathrm{pt}) \cong \mathcal{H}_{p+q}(X).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}_*^?$.
- We define a covariant functor called induction

$$\text{ind}\colon \mathcal{FGI} \to \Lambda\text{- Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}_n^G(\text{pt})$ and an injection of finite groups $\alpha\colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}_n^H(\text{pt}) \xrightarrow{\text{ind}_\alpha} \mathcal{H}_n^G(\text{ind}_\alpha \text{pt}) \xrightarrow{\mathcal{H}_n^G(\text{pr})} \mathcal{H}_n^G(\text{pt}).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}_*^?$.
- We define a covariant functor called induction

$$\text{ind} \colon \mathcal{FGI} \to \Lambda\text{- Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}_n^G(\text{pt})$ and an injection of finite groups $\alpha \colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}_n^H(\text{pt}) \xrightarrow{\text{ind}_\alpha} \mathcal{H}_n^G(\text{ind}_\alpha \text{pt}) \xrightarrow{\mathcal{H}_n^G(\text{pr})} \mathcal{H}_n^G(\text{pt}).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}_*^?$.
- We define a covariant functor called induction

$$\text{ind}\colon \mathcal{FGI} \to \Lambda\text{-}\operatorname{Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}_n^G(\text{pt})$ and an injection of finite groups $\alpha\colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}_n^H(\text{pt}) \xrightarrow{\text{ind}_\alpha} \mathcal{H}_n^G(\text{ind}_\alpha \text{pt}) \xrightarrow{\mathcal{H}_n^G(\text{pr})} \mathcal{H}_n^G(\text{pt}).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}^?_*$.
- We define a covariant functor called induction

$$\mathrm{ind}\colon \mathcal{FGI} \to \Lambda\text{-}\mathrm{Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}^G_n(\mathrm{pt})$ and an injection of finite groups $\alpha\colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}^H_n(\mathrm{pt}) \xrightarrow{\mathrm{ind}_\alpha} \mathcal{H}^G_n(\mathrm{ind}_\alpha \mathrm{pt}) \xrightarrow{\mathcal{H}^G_n(\mathrm{pr})} \mathcal{H}^G_n(\mathrm{pt}).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}^?_*$.
- We define a covariant functor called induction

$$\mathrm{ind}\colon \mathcal{FGI} \to \Lambda\text{-}\mathrm{Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}_n^G(\mathrm{pt})$ and an injection of finite groups $\alpha\colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}_n^H(\mathrm{pt}) \xrightarrow{\mathrm{ind}_\alpha} \mathcal{H}_n^G(\mathrm{ind}_\alpha \mathrm{pt}) \xrightarrow{\mathcal{H}_n^G(\mathrm{pr})} \mathcal{H}_n^G(\mathrm{pt}).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}^?_*$.
- We define a covariant functor called induction

$$\mathrm{ind}\colon \mathcal{FGI} \to \Lambda\text{-}\mathrm{Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}^G_n(\mathrm{pt})$ and an injection of finite groups $\alpha\colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}^H_n(\mathrm{pt}) \xrightarrow{\mathrm{ind}_\alpha} \mathcal{H}^G_n(\mathrm{ind}_\alpha \mathrm{pt}) \xrightarrow{\mathcal{H}^G_n(\mathrm{pr})} \mathcal{H}^G_n(\mathrm{pt}).$$

- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory $\mathcal{H}_*^?$.
- We define a covariant functor called induction

$$\mathrm{ind}\colon \mathcal{FGI} \to \Lambda\text{-}\mathrm{Mod}$$

from the category $\mathcal{FGI}$ of finite groups with injective group homomorphisms as morphisms to the category of Λ-modules as follows. It sends G to $\mathcal{H}_n^G(\mathrm{pt})$ and an injection of finite groups $\alpha\colon H \to G$ to the morphism given by the induction structure

$$\mathcal{H}_n^H(\mathrm{pt}) \xrightarrow{\mathrm{ind}_\alpha} \mathcal{H}_n^G(\mathrm{ind}_\alpha \mathrm{pt}) \xrightarrow{\mathcal{H}_n^G(\mathrm{pr})} \mathcal{H}_n^G(\mathrm{pt}).$$

Definition (Mackey extension)

We say that $\mathcal{H}_*^?$ has a Mackey extension if for every $n \in \mathbb{Z}$ there is a contravariant functor called restriction

$$\text{res}\colon \mathcal{FGI} \to \Lambda\text{- Mod}$$

such that these two functors ind and res agree on objects and satisfy the double coset formula ,i.e., we have for two subgroups $H, K \subset G$ of the finite group G

$$\text{res}_G^K \circ \text{ind}_H^G = \sum_{KgH \in K\backslash G/H} \text{ind}_{c(g)\colon H\cap g^{-1}Kg \to K} \circ \text{res}_H^{H\cap g^{-1}Kg},$$

where $c(g)$ is conjugation with g, i.e., $c(g)(h) = ghg^{-1}$.

Definition (Mackey extension)

We say that $\mathcal{H}_*^?$ has a Mackey extension if for every $n \in \mathbb{Z}$ there is a contravariant functor called restriction

$$\mathrm{res}\colon \mathcal{FGI} \to \Lambda\text{-}\mathrm{Mod}$$

such that these two functors ind and res agree on objects and satisfy the double coset formula ,i.e., we have for two subgroups $H, K \subset G$ of the finite group G

$$\mathrm{res}_G^K \circ \mathrm{ind}_H^G = \sum_{KgH \in K \backslash G/H} \mathrm{ind}_{c(g)\colon H\cap g^{-1}Kg \to K} \circ \mathrm{res}_H^{H\cap g^{-1}Kg},$$

where $c(g)$ is conjugation with g, i.e., $c(g)(h) = ghg^{-1}$.

Definition (Mackey extension)

We say that $\mathcal{H}_*^?$ has a Mackey extension if for every $n \in \mathbb{Z}$ there is a contravariant functor called restriction

$$\text{res}\colon \mathcal{FGI} \to \Lambda\text{-}\mathrm{Mod}$$

such that these two functors ind and res agree on objects and satisfy the double coset formula ,i.e., we have for two subgroups $H, K \subset G$ of the finite group G

$$\text{res}_G^K \circ \text{ind}_H^G = \sum_{KgH \in K\backslash G/H} \text{ind}_{c(g)\colon H \cap g^{-1}Kg \to K} \circ \text{res}_H^{H \cap g^{-1}Kg},$$

where $c(g)$ is conjugation with g, i.e., $c(g)(h) = ghg^{-1}$.

- In every case we will consider such a Mackey extension does exist and is given by an actual restriction.
- For instance for $H_0^?(-; \mathbf{K}^{\mathrm{top}})$ induction is the functor complex representation ring $R_{\mathbb{C}}$ with respect to induction of representations. The restriction part is given by the restriction of representations.

- In every case we will consider such a Mackey extension does exist and is given by an actual restriction.
- For instance for $H_0^?(-;\mathbf{K}^{\text{top}})$ induction is the functor complex representation ring $R_{\mathbb{C}}$ with respect to induction of representations. The restriction part is given by the restriction of representations.

- In every case we will consider such a Mackey extension does exist and is given by an actual restriction.
- For instance for $H_0^?(-;\mathbf{K}^{\mathrm{top}})$ induction is the functor complex representation ring $R_{\mathbb{C}}$ with respect to induction of representations. The restriction part is given by the restriction of representations.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.
Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H\backslash X^H;\Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- $C_G H$ is the centralizer and $N_G H$ the normalizer of $H \subseteq G$;
- $W_G H := N_G H / H \cdot C_G H$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \operatorname{cok}\left(\bigoplus_{\substack{K\subset H\\ K\neq H}} \operatorname{ind}_K^H \colon \bigoplus_{\substack{K\subset H\\ K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.

Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G : \bigoplus_{p+q=n} \bigoplus_{(H) \in I} H_p(C_G H \backslash X^H; \Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- $C_G H$ is the centralizer and $N_G H$ the normalizer of $H \subseteq G$;
- $W_G H := N_G H / H \cdot C_G H$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \mathrm{cok}\left(\bigoplus_{\substack{K \subset H \\ K \neq H}} \mathrm{ind}_K^H : \bigoplus_{\substack{K \subset H \\ K \neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.

Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H\backslash X^H;\Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- $C_G H$ is the centralizer and $N_G H$ the normalizer of $H \subseteq G$;
- $W_G H := N_G H / H \cdot C_G H$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \mathrm{cok}\left(\bigoplus_{\substack{K\subset H\\ K\neq H}} \mathrm{ind}_K^H \colon \bigoplus_{\substack{K\subset H\\ K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.

Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- C_GH is the centralizer and N_GH the normalizer of $H \subseteq G$;
- $W_GH := N_GH/H \cdot C_GH$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \mathrm{cok}\left(\bigoplus_{\substack{K\subset H\\K\neq H}} \mathrm{ind}_K^H \colon \bigoplus_{\substack{K\subset H\\K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right);$
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.

Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G\colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- C_GH is the centralizer and N_GH the normalizer of $H \subseteq G$;
- $W_GH := N_GH/H \cdot C_GH$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \operatorname{cok}\left(\bigoplus_{\substack{K\subset H\\ K\neq H}} \operatorname{ind}_K^H : \bigoplus_{\substack{K\subset H\\ K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.
Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H) \in I} H_p(C_G H \backslash X^H; \Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- $C_G H$ is the centralizer and $N_G H$ the normalizer of $H \subseteq G$;
- $W_G H := N_G H / H \cdot C_G H$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \operatorname{cok}\left(\bigoplus_{\substack{K \subset H \\ K \neq H}} \operatorname{ind}_K^H \colon \bigoplus_{\substack{K \subset H \\ K \neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.

Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- C_GH is the centralizer and N_GH the normalizer of $H \subseteq G$;
- $W_GH := N_GH/H \cdot C_GH$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \mathrm{cok}\left(\bigoplus_{\substack{K\subset H\\ K\neq H}} \mathrm{ind}_K^H \colon \bigoplus_{\substack{K\subset H\\ K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.
Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- C_GH is the centralizer and N_GH the normalizer of $H \subseteq G$;
- $W_GH := N_GH/H \cdot C_GH$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \operatorname{cok}\left(\bigoplus_{\substack{K\subset H\\ K\neq H}} \operatorname{ind}_K^H \colon \bigoplus_{\substack{K\subset H\\ K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.
Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- C_GH is the centralizer and N_GH the normalizer of $H \subseteq G$;
- $W_GH := N_GH/H \cdot C_GH$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \operatorname{cok}\left(\bigoplus_{\substack{K\subset H\\K\neq H}} \operatorname{ind}_K^H \colon \bigoplus_{\substack{K\subset H\\K\neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Equivariant Chern character, L. (2002))

Let $\mathcal{H}_*^?$ be a equivariant homology theory with values in Λ-modules for $\mathbb{Q} \subseteq \Lambda$. Suppose that $\mathcal{H}_*^?$ has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G.
Then there is for every group G, every proper G-CW-complex X and every $n \in \mathbb{Z}$ a natural isomorphism called equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H) \in I} H_p(C_G H \backslash X^H; \Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- $C_G H$ is the centralizer and $N_G H$ the normalizer of $H \subseteq G$;
- $W_G H := N_G H / H \cdot C_G H$ (This is always a finite group);
- $S_H\left(\mathcal{H}_q^H(*)\right) := \mathrm{cok}\left(\bigoplus_{\substack{K \subset H \\ K \neq H}} \mathrm{ind}_K^H \colon \bigoplus_{\substack{K \subset H \\ K \neq H}} \mathcal{H}_q^K(*) \to \mathcal{H}_q^H(*)\right)$;
- $\mathrm{ch}_*^?$ is an equivalence of equivariant homology theories.

Theorem (Artin's Theorem)

Let G be finite. Then the map

$$\bigoplus_{C \subset G} \operatorname{ind}_C^G \colon \bigoplus_{C \subset G} R_{\mathbb{C}}(C) \to R_{\mathbb{C}}(G)$$

is surjective after inverting $|G|$, where $C \subset G$ runs through the cyclic subgroups of G.

Let C be a finite cyclic group. The Artin defect is the cokernel of the map

$$\bigoplus_{D \subset C, D \neq C} \operatorname{ind}_D^C \colon \bigoplus_{D \subset C, D \neq C} R_{\mathbb{C}}(D) \to R_{\mathbb{C}}(C).$$

For an appropriate idempotent $\theta_C \in R_{\mathbb{Q}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right]$ the Artin defect is after inverting the order of $|C|$ canonically isomorphic to

$$\theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right].$$

Theorem (Artin's Theorem)

Let G be finite. Then the map

$$\bigoplus_{C\subset G} \operatorname{ind}_C^G : \bigoplus_{C\subset G} R_{\mathbb{C}}(C) \to R_{\mathbb{C}}(G)$$

is surjective after inverting $|G|$, where $C \subset G$ runs through the cyclic subgroups of G.

Let C be a finite cyclic group. The Artin defect is the cokernel of the map

$$\bigoplus_{D\subset C, D\neq C} \operatorname{ind}_D^C : \bigoplus_{D\subset C, D\neq C} R_{\mathbb{C}}(D) \to R_{\mathbb{C}}(C).$$

For an appropriate idempotent $\theta_C \in R_{\mathbb{Q}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right]$ the Artin defect is after inverting the order of $|C|$ canonically isomorphic to

$$\theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right].$$

Theorem (Artin's Theorem)

Let G be finite. Then the map

$$\bigoplus_{C\subset G} \operatorname{ind}_C^G : \bigoplus_{C\subset G} R_{\mathbb{C}}(C) \to R_{\mathbb{C}}(G)$$

is surjective after inverting $|G|$, where $C \subset G$ runs through the cyclic subgroups of G.

Let C be a finite cyclic group. The Artin defect is the cokernel of the map

$$\bigoplus_{D\subset C, D\neq C} \operatorname{ind}_D^C : \bigoplus_{D\subset C, D\neq C} R_{\mathbb{C}}(D) \to R_{\mathbb{C}}(C).$$

For an appropriate idempotent $\theta_C \in R_{\mathbb{Q}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right]$ the Artin defect is after inverting the order of $|C|$ canonically isomorphic to

$$\theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right].$$

Theorem (Artin's Theorem)

Let G be finite. Then the map

$$\bigoplus_{C \subset G} \operatorname{ind}_C^G : \bigoplus_{C \subset G} R_{\mathbb{C}}(C) \to R_{\mathbb{C}}(G)$$

is surjective after inverting $|G|$, where $C \subset G$ runs through the cyclic subgroups of G.

Let C be a finite cyclic group. The Artin defect is the cokernel of the map

$$\bigoplus_{D \subset C, D \neq C} \operatorname{ind}_D^C : \bigoplus_{D \subset C, D \neq C} R_{\mathbb{C}}(D) \to R_{\mathbb{C}}(C).$$

For an appropriate idempotent $\theta_C \in R_{\mathbb{Q}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right]$ the Artin defect is after inverting the order of $|C|$ canonically isomorphic to

$$\theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right].$$

Theorem (Artin's Theorem)

Let G be finite. Then the map

$$\bigoplus_{C \subset G} \operatorname{ind}_C^G : \bigoplus_{C \subset G} R_{\mathbb{C}}(C) \to R_{\mathbb{C}}(G)$$

is surjective after inverting $|G|$, where $C \subset G$ runs through the cyclic subgroups of G.

Let C be a finite cyclic group. The Artin defect is the cokernel of the map

$$\bigoplus_{D \subset C, D \neq C} \operatorname{ind}_D^C : \bigoplus_{D \subset C, D \neq C} R_{\mathbb{C}}(D) \to R_{\mathbb{C}}(C).$$

For an appropriate idempotent $\theta_C \in R_{\mathbb{Q}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right]$ the Artin defect is after inverting the order of $|C|$ canonically isomorphic to

$$\theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right].$$

Theorem (Artin's Theorem)

Let G be finite. Then the map

$$\bigoplus_{C \subset G} \operatorname{ind}_C^G \colon \bigoplus_{C \subset G} R_{\mathbb{C}}(C) \to R_{\mathbb{C}}(G)$$

is surjective after inverting $|G|$, where $C \subset G$ runs through the cyclic subgroups of G.

Let C be a finite cyclic group. The Artin defect is the cokernel of the map

$$\bigoplus_{D \subset C, D \neq C} \operatorname{ind}_D^C \colon \bigoplus_{D \subset C, D \neq C} R_{\mathbb{C}}(D) \to R_{\mathbb{C}}(C).$$

For an appropriate idempotent $\theta_C \in R_{\mathbb{Q}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right]$ the Artin defect is after inverting the order of $|C|$ canonically isomorphic to

$$\theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|C|}\right].$$

- Let $K_*^G = H_*^?(-; \mathbf{K}^{\text{top}})$ be equivariant topological K-theory.
- We get for a finite subgroup $H \subseteq G$

$$K_n^G(G/H) = K_n^H(\text{pt}) = \begin{cases} R_{\mathbb{C}}(H) & \text{if } n \text{ is even;} \\ \{0\} & \text{if } n \text{ is odd.} \end{cases}$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- Let $K_*^G = H_*^?(-;\mathbf{K}^{\text{top}})$ be equivariant topological K-theory.
- We get for a finite subgroup $H \subseteq G$

$$K_n^G(G/H) = K_n^H(\text{pt}) = \begin{cases} R_{\mathbb{C}}(H) & \text{if } n \text{ is even;} \\ \{0\} & \text{if } n \text{ is odd.} \end{cases}$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- Let $K_*^G = H_*^?(-;\mathbf{K}^{\mathrm{top}})$ be equivariant topological K-theory.
- We get for a finite subgroup $H \subseteq G$

$$K_n^G(G/H) = K_n^H(\mathrm{pt}) = \begin{cases} R_{\mathbb{C}}(H) & \text{if } n \text{ is even;} \\ \{0\} & \text{if } n \text{ is odd.} \end{cases}$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- Let $K_*^G = H_*^?(-;\mathbf{K}^{\text{top}})$ be equivariant topological K-theory.
- We get for a finite subgroup $H \subseteq G$

$$K_n^G(G/H) = K_n^H(\text{pt}) = \begin{cases} R_{\mathbb{C}}(H) & \text{if } n \text{ is even;} \\ \{0\} & \text{if } n \text{ is odd.} \end{cases}$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- Recall

$$\mathrm{ch}_n^G\colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

Example (Improvement of Artin's Theorem)

Let G be finite, $X = \{*\}$ and $\mathcal{H}_*^? = K_*^?$. Then we get an improvement of Artin's theorem. Namely, the equivariant Chern character induces an isomorphism

$$\mathrm{ch}_0^G(\mathrm{pt})\colon \bigoplus_{(C)} \mathbb{Z} \otimes_{\mathbb{Z}[W_GC]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right] \xrightarrow{\cong} R_{\mathbb{C}}(G) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right]$$

where (C) runs over the conjugacy classes of finite cyclic subgroups.

- Recall

$$\mathrm{ch}_n^G\colon \bigoplus_{p+q=n}\bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda)\otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right)\xrightarrow{\cong}\mathcal{H}_n^G(X).$$

Example (Improvement of Artin's Theorem)

Let G be finite, $X=\{*\}$ and $\mathcal{H}_*^? = K_*^?$. Then we get an improvement of Artin's theorem. Namely, the equivariant Chern character induces an isomorphism

$$\mathrm{ch}_0^G(\mathrm{pt})\colon \bigoplus_{(C)}\mathbb{Z}\otimes_{\mathbb{Z}[W_GC]}\theta_C\cdot R_{\mathbb{C}}(C)\otimes_{\mathbb{Z}}\mathbb{Z}\left[\frac{1}{|G|}\right]\xrightarrow{\cong} R_{\mathbb{C}}(G)\otimes_{\mathbb{Z}}\mathbb{Z}\left[\frac{1}{|G|}\right]$$

where (C) runs over the conjugacy classes of finite cyclic subgroups.

- Recall

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H\backslash X^H;\Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

Example (Improvement of Artin's Theorem)

Let G be finite, $X = \{*\}$ and $\mathcal{H}_*^? = K_*^?$. Then we get an improvement of Artin's theorem. Namely, the equivariant Chern character induces an isomorphism

$$\mathrm{ch}_0^G(\mathrm{pt}) \colon \bigoplus_{(C)} \mathbb{Z} \otimes_{\mathbb{Z}[W_G C]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right] \xrightarrow{\cong} R_{\mathbb{C}}(G) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right]$$

where (C) runs over the conjugacy classes of finite cyclic subgroups.

- Recall

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H\backslash X^H;\Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

Example (Improvement of Artin's Theorem)

Let G be finite, $X = \{*\}$ and $\mathcal{H}_*^? = K_*^?$. Then we get an improvement of Artin's theorem. Namely, the equivariant Chern character induces an isomorphism

$$\mathrm{ch}_0^G(\mathrm{pt}) \colon \bigoplus_{(C)} \mathbb{Z} \otimes_{\mathbb{Z}[W_G C]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right] \xrightarrow{\cong} R_{\mathbb{C}}(G) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right]$$

where (C) runs over the conjugacy classes of finite cyclic subgroups.

- Recall

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H \backslash X^H; \Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

Example (Improvement of Artin's Theorem)

Let G be finite, $X = \{*\}$ and $\mathcal{H}_*^? = K_*^?$. Then we get an improvement of Artin's theorem. Namely, the equivariant Chern character induces an isomorphism

$$\mathrm{ch}_0^G(\mathrm{pt}) \colon \bigoplus_{(C)} \mathbb{Z} \otimes_{\mathbb{Z}[W_G C]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right] \xrightarrow{\cong} R_{\mathbb{C}}(G) \otimes_{\mathbb{Z}} \mathbb{Z}\left[\frac{1}{|G|}\right]$$

where (C) runs over the conjugacy classes of finite cyclic subgroups.

Corollary (Rational computation of $K_*^G(\underline{E}G)$)

For every group G and every $n \in \mathbb{Z}$ we obtain an isomorphism

$$\bigoplus_{(C)} \bigoplus_{k} H_{p+2k}(BC_GC) \otimes_{\mathbb{Z}[W_GC]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} K_n^G(\underline{E}G) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- If the Baum-Connes Conjecture holds for G, this gives a computation of $K_n(C_r^*(G)) \otimes_{\mathbb{Z}} \mathbb{Q}$.

Corollary (Rational computation of $K_*^G(\underline{E}G)$)

For every group G and every $n \in \mathbb{Z}$ we obtain an isomorphism

$$\bigoplus_{(C)} \bigoplus_{k} H_{p+2k}(BC_GC) \otimes_{\mathbb{Z}[W_GC]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} K_n^G(\underline{E}G) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- If the Baum-Connes Conjecture holds for G, this gives a computation of $K_n(C_r^*(G)) \otimes_{\mathbb{Z}} \mathbb{Q}$.

Corollary (Rational computation of $K_*^G(\underline{E}G)$)

For every group G and every $n \in \mathbb{Z}$ we obtain an isomorphism

$$\bigoplus_{(C)} \bigoplus_{k} H_{p+2k}(BC_GC) \otimes_{\mathbb{Z}[W_GC]} \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} K_n^G(\underline{E}G) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- If the Baum-Connes Conjecture holds for G, this gives a computation of $K_n(C_r^*(G)) \otimes_{\mathbb{Z}} \mathbb{Q}$.

- The last corollary follows from the equivariant Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X)$$

using the following facts.

- $\underline{E}G^C$ is a contractible proper C_GC- space. Hence the canonical map $BC_GC \to C_GC\backslash\underline{E}G^C$ induces an isomorphism

$$H_p(BC_GC) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} H_p(C_GC\backslash\underline{E}G^C) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- The last corollary follows from the equivariant Chern character

$$\mathrm{ch}_n^G\colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H\backslash X^H;\Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X)$$

using the following facts.

- $\underline{E}G^C$ is a contractible proper $C_G C$- space. Hence the canonical map $BC_G C \to C_G C\backslash \underline{E}G^C$ induces an isomorphism

$$H_p(BC_G C) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} H_p(C_G C\backslash \underline{E}G^C) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- The last corollary follows from the equivariant Chern character

$$\mathrm{ch}_n^G\colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X)$$

using the following facts.

- $\underline{E}G^C$ is a contractible proper C_GC- space. Hence the canonical map $BC_GC \to C_GC\backslash\underline{E}G^C$ induces an isomorphism

$$H_p(BC_GC) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} H_p(C_GC\backslash\underline{E}G^C) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- The last corollary follows from the equivalent Chern character

$$\mathrm{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X)$$

  using the following facts.
- $\underline{E}G^C$ is a contractible proper C_GC- space. Hence the canonical map $BC_GC \to C_GC\backslash\underline{E}G^C$ induces an isomorphism

$$H_p(BC_GC) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} H_p(C_GC\backslash\underline{E}G^C) \otimes_{\mathbb{Z}} \mathbb{Q}.$$

- $S_H\left(K_q^H(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K_q^C(*)\right) \otimes_{\mathbb{Z}} \mathbb{Q} = \theta_C \cdot R_{\mathbb{C}}(C) \otimes_{\mathbb{Z}} \mathbb{Q}$ if C is finite cyclic and q is even.

- The last corollary follows from the equivariant Chern character

$$\mathrm{ch}^G_n\colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_GH\backslash X^H;\Lambda) \otimes_{\Lambda[W_GH]} S_H\left(\mathcal{H}^H_q(*)\right) \xrightarrow{\cong} \mathcal{H}^G_n(X)$$

  using the following facts.
- $\underline{E}G^C$ is a contractible proper C_GC- space. Hence the canonical map $BC_GC \to C_GC\backslash\underline{E}G^C$ induces an isomorphism

$$H_p(BC_GC)\otimes_{\mathbb{Z}}\mathbb{Q} \xrightarrow{\cong} H_p(C_GC\backslash\underline{E}G^C)\otimes_{\mathbb{Z}}\mathbb{Q}.$$

- $S_H\left(K^H_q(*)\right)\otimes_{\mathbb{Z}}\mathbb{Q} = 0$ if H is not cyclic and q is even or if q is odd.
- $S_C\left(K^C_q(*)\right)\otimes_{\mathbb{Z}}\mathbb{Q} = \theta_C\cdot R_{\mathbb{C}}(C)\otimes_{\mathbb{Z}}\mathbb{Q}$ if C is finite cyclic and q is even.

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\mathrm{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\mathrm{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\text{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\operatorname{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\operatorname{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group. Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\operatorname{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\mathrm{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$K^0(BG) \cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)};$$

$$K^1(BG) \cong 0.$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\operatorname{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$
\begin{aligned}
K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\
&\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\
K^1(BG) &\cong 0.
\end{aligned}
$$

Topological K-theory of classifying spaces

- For a prime p denote by $r(p) = |\mathrm{con}_p(G)|$ the number of conjugacy classes (g) of elements $g \neq 1$ in G of p-power order.
- $\mathbb{I}_G$ is the augmentation ideal of $R_{\mathbb{C}}(G)$.
- Let $\mathbb{I}_p(G)$ be the image of the restriction homomorphism $\mathbb{I}(G) \to \mathbb{I}(G_p)$.

Theorem (Completion Theorem, Atiyah-Segal (1969))

Let G be a finite group.
Then there are isomorphisms of abelian groups

$$\begin{aligned} K^0(BG) &\cong R_{\mathbb{C}}(G)\widehat{_{\mathbb{I}_G}} \\ &\cong \mathbb{Z} \times \prod_{p \text{ prime}} \mathbb{I}_p(G) \otimes_{\mathbb{Z}} \widehat{\mathbb{Z}_p} \cong \mathbb{Z} \times \prod_{p \text{ prime}} (\widehat{\mathbb{Z}_p})^{r(p)}; \\ K^1(BG) &\cong 0. \end{aligned}$$

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.
Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}_G^n \colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g\rangle;\widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact G-CW-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.
Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}^n_G\colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g\rangle;\widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact *G-CW*-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.

Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}_G^n \colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g\rangle;\widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact G-CW-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.

Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}_G^n \colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g\rangle;\widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact G-CW-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.

Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}^n_G \colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g \rangle;\widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact G-CW-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.
Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}_G^n \colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g \rangle; \widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact G-CW-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Theorem (L. (2005))

Let G be a discrete group. Denote by $K^*(BG)$ the topological (complex) K-theory of its classifying space BG. Suppose that there is a cocompact G-CW-model for the classifying space $\underline{E}G$ for proper G-actions.

Then there is a $\mathbb{Q}$-isomorphism

$$\overline{\mathrm{ch}}_G^n \colon K^n(BG) \otimes_{\mathbb{Z}} \mathbb{Q} \xrightarrow{\cong} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BG;\mathbb{Q})\right) \times \prod_{p \text{ prime}} \prod_{(g) \in \mathrm{con}_p(G)} \left(\prod_{i \in \mathbb{Z}} H^{2i+n}(BC_G\langle g\rangle;\widehat{\mathbb{Q}_p})\right).$$

- The multiplicative structure can also be determined.
- There are many groups for which a cocompact G-CW-model for $\underline{E}G$ exists, e.g., hyperbolic groups.

Example ($SL_3(\mathbb{Z})$)

- It is well-known that its rational cohomology satisfies $\widetilde{H}^n(BSL_3(\mathbb{Z});\mathbb{Q}) = 0$ for all $n \in \mathbb{Z}$.
- Actually, by a result of Soule (1978) the quotient space $SL_3(\mathbb{Z})\backslash \underline{E}SL_3(\mathbb{Z})$ is contractible and compact.
- From the classification of finite subgroups of $SL_3(\mathbb{Z})$ we see that $SL_3(\mathbb{Z})$ contains up to conjugacy two elements of order 2, two elements of order 4 and two elements of order 3 and no further conjugacy classes of non-trivial elements of prime power order.
- The rational homology of each of the centralizers of elements in $\mathrm{con}_2(G)$ and $\mathrm{con}_3(G)$ agrees with the one of the trivial group.
- Hence we get

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong \mathbb{Q} \times (\mathbb{Q}\widehat{_2})^4 \times (\mathbb{Q}\widehat{_3})^2; \\ K^1(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong 0. \end{aligned}$$

Example ($SL_3(\mathbb{Z})$)

- It is well-known that its rational cohomology satisfies $\widetilde{H}^n(BSL_3(\mathbb{Z});\mathbb{Q}) = 0$ for all $n \in \mathbb{Z}$.
- Actually, by a result of Soule (1978) the quotient space $SL_3(\mathbb{Z})\backslash \underline{E}SL_3(\mathbb{Z})$ is contractible and compact.
- From the classification of finite subgroups of $SL_3(\mathbb{Z})$ we see that $SL_3(\mathbb{Z})$ contains up to conjugacy two elements of order 2, two elements of order 4 and two elements of order 3 and no further conjugacy classes of non-trivial elements of prime power order.
- The rational homology of each of the centralizers of elements in $\operatorname{con}_2(G)$ and $\operatorname{con}_3(G)$ agrees with the one of the trivial group.
- Hence we get

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong \mathbb{Q} \times (\mathbb{Q}\widehat{_2})^4 \times (\mathbb{Q}\widehat{_3})^2; \\ K^1(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong 0. \end{aligned}$$

Example ($SL_3(\mathbb{Z})$)

- It is well-known that its rational cohomology satisfies $\widetilde{H}^n(BSL_3(\mathbb{Z});\mathbb{Q}) = 0$ for all $n \in \mathbb{Z}$.
- Actually, by a result of Soule (1978) the quotient space $SL_3(\mathbb{Z})\backslash \underline{E}SL_3(\mathbb{Z})$ is contractible and compact.
- From the classification of finite subgroups of $SL_3(\mathbb{Z})$ we see that $SL_3(\mathbb{Z})$ contains up to conjugacy two elements of order 2, two elements of order 4 and two elements of order 3 and no further conjugacy classes of non-trivial elements of prime power order.
- The rational homology of each of the centralizers of elements in $\mathrm{con}_2(G)$ and $\mathrm{con}_3(G)$ agrees with the one of the trivial group.
- Hence we get

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong \mathbb{Q} \times (\mathbb{Q}\widehat{_2})^4 \times (\mathbb{Q}\widehat{_3})^2; \\ K^1(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong 0. \end{aligned}$$

Example ($SL_3(\mathbb{Z})$)

- It is well-known that its rational cohomology satisfies $\widetilde{H}^n(BSL_3(\mathbb{Z});\mathbb{Q}) = 0$ for all $n \in \mathbb{Z}$.
- Actually, by a result of Soule (1978) the quotient space $SL_3(\mathbb{Z})\backslash \underline{E}SL_3(\mathbb{Z})$ is contractible and compact.
- From the classification of finite subgroups of $SL_3(\mathbb{Z})$ we see that $SL_3(\mathbb{Z})$ contains up to conjugacy two elements of order 2, two elements of order 4 and two elements of order 3 and no further conjugacy classes of non-trivial elements of prime power order.
- The rational homology of each of the centralizers of elements in $\mathrm{con}_2(G)$ and $\mathrm{con}_3(G)$ agrees with the one of the trivial group.
- Hence we get

$$
\begin{aligned}
K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong \mathbb{Q} \times (\mathbb{Q}\widehat{_2})^4 \times (\mathbb{Q}\widehat{_3})^2; \\
K^1(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong 0.
\end{aligned}
$$

Example ($SL_3(\mathbb{Z})$)

- It is well-known that its rational cohomology satisfies $\widetilde{H}^n(BSL_3(\mathbb{Z});\mathbb{Q}) = 0$ for all $n \in \mathbb{Z}$.
- Actually, by a result of Soule (1978) the quotient space $SL_3(\mathbb{Z})\backslash \underline{E}SL_3(\mathbb{Z})$ is contractible and compact.
- From the classification of finite subgroups of $SL_3(\mathbb{Z})$ we see that $SL_3(\mathbb{Z})$ contains up to conjugacy two elements of order 2, two elements of order 4 and two elements of order 3 and no further conjugacy classes of non-trivial elements of prime power order.
- The rational homology of each of the centralizers of elements in $\mathrm{con}_2(G)$ and $\mathrm{con}_3(G)$ agrees with the one of the trivial group.
- Hence we get

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong \mathbb{Q} \times (\mathbb{Q}\widehat{_2})^4 \times (\mathbb{Q}\widehat{_3})^2; \\ K^1(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong 0. \end{aligned}$$

Example ($SL_3(\mathbb{Z})$)

- It is well-known that its rational cohomology satisfies $\widetilde{H}^n(BSL_3(\mathbb{Z});\mathbb{Q}) = 0$ for all $n \in \mathbb{Z}$.
- Actually, by a result of Soule (1978) the quotient space $SL_3(\mathbb{Z})\backslash \underline{E}SL_3(\mathbb{Z})$ is contractible and compact.
- From the classification of finite subgroups of $SL_3(\mathbb{Z})$ we see that $SL_3(\mathbb{Z})$ contains up to conjugacy two elements of order 2, two elements of order 4 and two elements of order 3 and no further conjugacy classes of non-trivial elements of prime power order.
- The rational homology of each of the centralizers of elements in $\text{con}_2(G)$ and $\text{con}_3(G)$ agrees with the one of the trivial group.
- Hence we get

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong \mathbb{Q} \times (\mathbb{Q}\widehat{_2})^4 \times (\mathbb{Q}\widehat{_3})^2; \\ K^1(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong 0. \end{aligned}$$

Example (Continued)

- The identification of $K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q}$ above is compatible with the multiplicative structures.
- Actually the computation using Brown-Petersen cohomology and the Conner-Floyd relation by Tezuka-Yagita (1992) gives the integral computation

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) &\cong \mathbb{Z} \times (\widehat{\mathbb{Z}_2})^4 \times (\widehat{\mathbb{Z}_3})^2; \\ K^1(BSL_3(\mathbb{Z})) &\cong 0. \end{aligned}$$

- Soule (1978) has computed the integral cohomology of $SL_3(\mathbb{Z})$.

Example (Continued)

- The identification of $K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q}$ above is compatible with the multiplicative structures.
- Actually the computation using Brown-Petersen cohomology and the Conner-Floyd relation by Tezuka-Yagita (1992) gives the integral computation

$$\begin{aligned} K^0(BSL_3(\mathbb{Z})) &\cong \mathbb{Z} \times (\widehat{\mathbb{Z}_2})^4 \times (\widehat{\mathbb{Z}_3})^2; \\ K^1(BSL_3(\mathbb{Z})) &\cong 0. \end{aligned}$$

- Soule (1978) has computed the integral cohomology of $SL_3(\mathbb{Z})$.

Example (Continued)

- The identification of $K^0(BSL_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q}$ above is compatible with the multiplicative structures.
- Actually the computation using Brown-Petersen cohomology and the Conner-Floyd relation by Tezuka-Yagita (1992) gives the integral computation

$$
\begin{aligned}
K^0(BSL_3(\mathbb{Z})) &\cong \mathbb{Z} \times (\widehat{\mathbb{Z}_2})^4 \times (\widehat{\mathbb{Z}_3})^2; \\
K^1(BSL_3(\mathbb{Z})) &\cong 0.
\end{aligned}
$$

- Soule (1978) has computed the integral cohomology of $SL_3(\mathbb{Z})$.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \Rightarrow N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \Rightarrow N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \;\Rightarrow\; N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:

  (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;

  (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \Rightarrow N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \Rightarrow N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \Rightarrow N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \;\Rightarrow\; N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \Rightarrow N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \;\Rightarrow\; N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

A computation

- Let G be a discrete group. Let $\mathcal{MFIN}$ be the subset of $\mathcal{FIN}$ consisting of elements in $\mathcal{FIN}$ which are maximal in $\mathcal{FIN}$.
- Assume that G satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
  - (NM) $M \in \mathcal{MFIN}, M \neq \{1\} \;\Rightarrow\; N_G M = M$.
- Here are some examples of groups G which satisfy conditions (M) and (NM):
  - Extensions $1 \to \mathbb{Z}^n \to G \to F \to 1$ for finite F such that the conjugation action of F on $\mathbb{Z}^n$ is free outside $0 \in \mathbb{Z}^n$;
  - Fuchsian groups;
  - One-relator groups G.

- For such a group there is a nice model for $\underline{E}G$ with as few non-free cells as possible. Let $\{(M_i) \mid i \in I\}$ be the set of conjugacy classes of maximal finite subgroups of $M_i \subseteq G$. By attaching free G-cells we get an inclusion of G-CW-complexes $j_1\colon \coprod_{i \in I} G \times_{M_i} EM_i \to EG$.
- Define $\underline{E}G$ as the G-pushout

$$\begin{array}{ccc} \coprod_{i \in I} G \times_{M_i} EM_i & \xrightarrow{j_1} & EG \\ \downarrow{u_1} & & \downarrow{f_1} \\ \coprod_{i \in I} G/M_i & \xrightarrow{k_1} & \underline{E}G \end{array}$$

where u_1 is the obvious G-map obtained by collapsing each EM_i to a point.

- For such a group there is a nice model for $\underline{E}G$ with as few non-free cells as possible. Let $\{(M_i) \mid i \in I\}$ be the set of conjugacy classes of maximal finite subgroups of $M_i \subseteq G$. By attaching free G-cells we get an inclusion of G-CW-complexes $j_1 \colon \coprod_{i \in I} G \times_{M_i} EM_i \to EG$.
- Define $\underline{E}G$ as the G-pushout

$$\begin{array}{ccc} \coprod_{i \in I} G \times_{M_i} EM_i & \xrightarrow{j_1} & EG \\ \big\downarrow u_1 & & \big\downarrow f_1 \\ \coprod_{i \in I} G/M_i & \xrightarrow{k_1} & \underline{E}G \end{array}$$

where u_1 is the obvious G-map obtained by collapsing each EM_i to a point.

- For such a group there is a nice model for $\underline{E}G$ with as few non-free cells as possible. Let $\{(M_i) \mid i \in I\}$ be the set of conjugacy classes of maximal finite subgroups of $M_i \subseteq G$. By attaching free G-cells we get an inclusion of G-CW-complexes $j_1 \colon \coprod_{i \in I} G \times_{M_i} EM_i \to EG$.
- Define $\underline{E}G$ as the G-pushout

$$\begin{array}{ccc} \coprod_{i \in I} G \times_{M_i} EM_i & \xrightarrow{j_1} & EG \\ \downarrow u_1 & & \downarrow f_1 \\ \coprod_{i \in I} G/M_i & \xrightarrow{k_1} & \underline{E}G \end{array}$$

where u_1 is the obvious G-map obtained by collapsing each EM_i to a point.

- For such a group there is a nice model for $\underline{E}G$ with as few non-free cells as possible. Let $\{(M_i) \mid i \in I\}$ be the set of conjugacy classes of maximal finite subgroups of $M_i \subseteq G$. By attaching free G-cells we get an inclusion of G-CW-complexes $j_1 \colon \coprod_{i \in I} G \times_{M_i} EM_i \to EG$.
- Define $\underline{E}G$ as the G-pushout

$$\begin{array}{ccc} \coprod_{i \in I} G \times_{M_i} EM_i & \xrightarrow{j_1} & EG \\ \downarrow u_1 & & \downarrow f_1 \\ \coprod_{i \in I} G/M_i & \xrightarrow{k_1} & \underline{E}G \end{array}$$

where u_1 is the obvious G-map obtained by collapsing each EM_i to a point.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = (G/M_{i_0})^H = \text{pt}.$$

Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt.}$$

Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

Hence $\underline{E}G^H = \text{pt}$.

- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

Hence $\underline{E}G^H = \text{pt}$.

- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \mathrm{pt}.$$

Hence $\underline{E}G^H = \mathrm{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

  Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Next we explain why $\underline{E}G$ is a model for the classifying space for proper actions of G.
- Its isotropy groups are all finite. We have to show for $H \subseteq G$ finite that $\underline{E}G^H$ contractible.
- We begin with the case $H \neq \{1\}$. Because of conditions (M) and (NM) there is precisely one index $i_0 \in I$ such that H is subconjugated to M_{i_0} and is not subconjugated to M_i for $i \neq i_0$. We get

$$\left(\coprod_{i \in I} G/M_i\right)^H = \left(G/M_{i_0}\right)^H = \text{pt}.$$

  Hence $\underline{E}G^H = \text{pt}$.
- It remains to treat $H = \{1\}$. Since u_1 is a non-equivariant homotopy equivalence and j_1 is a cofibration, f_1 is a non-equivariant homotopy equivalence. Hence $\underline{E}G$ is contractible.

- Consider the pushout obtained from the G-pushout above by dividing the G-action

$$\begin{array}{ccc} \coprod_{i\in I} BM_i & \longrightarrow & BG \\ \downarrow & & \downarrow \\ \coprod_{i\in I} \mathrm{pt} & \longrightarrow & G\backslash \underline{E}G \end{array}$$

- The associated Mayer-Vietoris sequence yields

$$\ldots \to \widetilde{H}_{p+1}(G\backslash \underline{E}G) \to \bigoplus_{i\in I} \widetilde{H}_p(BM_i) \to \widetilde{H}_p(BG) \to \widetilde{H}_p(G\backslash \underline{E}G) \to \ldots$$

- In particular we obtain an isomorphism for $p \geq \dim(\underline{E}G) + 2$

$$\bigoplus_{i\in I} H_p(BM_i) \xrightarrow{\cong} H_p(BG).$$

- Consider the pushout obtained from the G-pushout above by dividing the G-action

$$\begin{array}{ccc} \coprod_{i \in I} BM_i & \longrightarrow & BG \\ \downarrow & & \downarrow \\ \coprod_{i \in I} \mathrm{pt} & \longrightarrow & G\backslash \underline{E}G \end{array}$$

- The associated Mayer-Vietoris sequence yields

$$\ldots \to \widetilde{H}_{p+1}(G\backslash \underline{E}G) \to \bigoplus_{i \in I} \widetilde{H}_p(BM_i) \to \widetilde{H}_p(BG) \to \widetilde{H}_p(G\backslash \underline{E}G) \to \ldots$$

- In particular we obtain an isomorphism for $p \geq \dim(\underline{E}G) + 2$

$$\bigoplus_{i \in I} H_p(BM_i) \xrightarrow{\cong} H_p(BG).$$

- Consider the pushout obtained from the G-pushout above by dividing the G-action

$$\begin{array}{ccc} \coprod_{i \in I} BM_i & \longrightarrow & BG \\ \downarrow & & \downarrow \\ \coprod_{i \in I} \mathrm{pt} & \longrightarrow & G\backslash \underline{E}G \end{array}$$

- The associated Mayer-Vietoris sequence yields

$$\ldots \to \widetilde{H}_{p+1}(G\backslash \underline{E}G) \to \bigoplus_{i \in I} \widetilde{H}_p(BM_i) \to \widetilde{H}_p(BG) \to \widetilde{H}_p(G\backslash \underline{E}G) \to \ldots$$

- In particular we obtain an isomorphism for $p \geq \dim(\underline{E}G) + 2$

$$\bigoplus_{i \in I} H_p(BM_i) \xrightarrow{\cong} H_p(BG).$$

Theorem

Let G be a discrete group which satisfies the conditions (M) and (NM) above.

Then there is an isomorphism

$$K_1^G(\underline{E}G) \xrightarrow{\cong} K_1(G\backslash\underline{E}G),$$

and a short exact sequence

$$0 \to \bigoplus_{i \in I} \widetilde{R}_{\mathbb{C}}(M_i) \to K_0(\underline{E}G) \to K_0(G\backslash\underline{E}G) \to 0.$$

It splits if we invert the orders of all finite subgroups of G.

- If the Baum-Connes Conjecture is true for G, then

$$K_n(C_r^*(G)) \cong K_n^G(\underline{E}G).$$

Theorem

Let G be a discrete group which satisfies the conditions (M) and (NM) above.
Then there is an isomorphism

$$K_1^G(\underline{E}G) \xrightarrow{\cong} K_1(G\backslash\underline{E}G),$$

and a short exact sequence

$$0 \to \bigoplus_{i \in I} \widetilde{R}_{\mathbb{C}}(M_i) \to K_0(\underline{E}G) \to K_0(G\backslash\underline{E}G) \to 0.$$

It splits if we invert the orders of all finite subgroups of G.

- If the Baum-Connes Conjecture is true for G, then

$$K_n(C_r^*(G)) \cong K_n^G(\underline{E}G).$$

Theorem

Let G be a discrete group which satisfies the conditions (M) and (NM) above.
Then there is an isomorphism

$$K_1^G(\underline{E}G) \xrightarrow{\cong} K_1(G\backslash\underline{E}G),$$

and a short exact sequence

$$0 \to \bigoplus_{i \in I} \widetilde{R}_{\mathbb{C}}(M_i) \to K_0(\underline{E}G) \to K_0(G\backslash\underline{E}G) \to 0.$$

It splits if we invert the orders of all finite subgroups of G.

- If the Baum-Connes Conjecture is true for G, then

$$K_n(C_r^*(G)) \cong K_n^G(\underline{E}G).$$

Theorem

Let G be a discrete group which satisfies the conditions (M) and (NM) above.
Then there is an isomorphism

$$K_1^G(\underline{E}G) \xrightarrow{\cong} K_1(G\backslash\underline{E}G),$$

and a short exact sequence

$$0 \to \bigoplus_{i \in I} \widetilde{R}_{\mathbb{C}}(M_i) \to K_0(\underline{E}G) \to K_0(G\backslash\underline{E}G) \to 0.$$

It splits if we invert the orders of all finite subgroups of G.

- If the Baum-Connes Conjecture is true for G, then

$$K_n(C_r^*(G)) \cong K_n^G(\underline{E}G).$$

Theorem

Let G be a discrete group which satisfies the conditions (M) and (NM) above.
Then there is an isomorphism

$$K_1^G(\underline{E}G) \xrightarrow{\cong} K_1(G\backslash\underline{E}G),$$

and a short exact sequence

$$0 \to \bigoplus_{i \in I} \widetilde{R}_{\mathbb{C}}(M_i) \to K_0(\underline{E}G) \to K_0(G\backslash\underline{E}G) \to 0.$$

It splits if we invert the orders of all finite subgroups of G.

- If the Baum-Connes Conjecture is true for G, then

$$K_n(C_r^*(G)) \cong K_n^G(\underline{E}G).$$

- We see that for computations of group homology or of K- and L-groups of group rings and group C^*-algebras it is important to understand the spaces $G\backslash \underline{E}G$.
- Often geometric input ensures that $G\backslash \underline{E}G$ is a fairly small CW-complex.
- On the other hand recall the result due to Leary-Nucinkis (2001) that for any CW-complex X there exists a group G with $X \simeq G\backslash \underline{E}G$.

- We see that for computations of group homology or of K- and L-groups of group rings and group C^*-algebras it is important to understand the spaces $G\backslash \underline{E}G$.
- Often geometric input ensures that $G\backslash \underline{E}G$ is a fairly small CW-complex.
- On the other hand recall the result due to Leary-Nucinkis (2001) that for any CW-complex X there exists a group G with $X \simeq G\backslash \underline{E}G$.

- We see that for computations of group homology or of K- and L-groups of group rings and group C^*-algebras it is important to understand the spaces $G\backslash \underline{E}G$.
- Often geometric input ensures that $G\backslash \underline{E}G$ is a fairly small CW-complex.
- On the other hand recall the result due to Leary-Nucinkis (2001) that for any CW-complex X there exists a group G with $X \simeq G\backslash \underline{E}G$.

Question (Consequences)

What are the consequences of the Farrell-Jones Conjecture and the Baum-Connes Conjecture?

To be continued

Stay tuned

Question (Consequences)

What are the consequences of the Farrell-Jones Conjecture and the Baum-Connes Conjecture?

To be continued

Stay tuned

Question (Consequences)

What are the consequences of the Farrell-Jones Conjecture and the Baum-Connes Conjecture?

To be continued

Stay tuned

Question (Consequences)

What are the consequences of the Farrell-Jones Conjecture and the Baum-Connes Conjecture?

To be continued

Stay tuned