# Equivariant homology theories (Lecture IV)

Wolfgang Lück Münster Germany email lueck@math.uni-muenster.de http://www.math.uni-muenster.de/u/lueck/

Hangzhou, July 2007

- We have introduced the Farrell-Jones Conjecture and the Baum-Connes Conjecture for torsionfree groups and discussed applications of these conjectures such as to the Kaplansky Conjecture and the Borel Conjecture.
- We have explained that the formulations for torsionfree groups cannot extend to arbitrary groups.
   Our goal is to find a formulation which makes sense for all groups and all rings.
- For this purpose we have introduced classifying spaces for families of subgroups of a group *G* which we will recall next.
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Examples for  $\mathcal{F}$  are:

- $T\mathcal{R} = {\text{trivial subgroup}};$
- $\mathcal{FIN} = \{ \text{finite subgroups} \};$
- $\mathcal{FCYC} = \{ \text{finite cyclic subgroups} \};$
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Let  $\mathcal{F}$  be a family of subgroups of G. A model for the *classifying G-CW-complex for the family*  $\mathcal{F}$  is a *G-CW*-complex  $E_{\mathcal{F}}(G)$  which has the following properties:

- All isotropy groups of  $E_{\mathcal{F}}(G)$  belong to  $\mathcal{F}$ ;
- For any *G*-*CW*-complex *Y*, whose isotropy groups belong to  $\mathcal{F}$ , there is up to *G*-homotopy precisely one *G*-map  $Y \rightarrow X$ .

We abbreviate  $\underline{E}G := E_{\mathcal{FIN}}(G)$  and call it the *universal G-CW-complex for proper G-actions*. We also write  $EG = E_{\mathcal{FP}}(G)$ 

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Can nice geometric models for <u>E</u>G be used to compute the group homology and more general homology and cohomology theories of a group G?

Question (*K*-theory of group rings and group homology)

Is there a relation between  $K_n(RG)$  and the group homology of G?

Question (Isomorphism Conjectures and classifying spaces of families)

Can classifying spaces of families be used to formulate a version of the Farrell-Jones Conjecture and the Baum-Connes Conjecture which may hold for all groups and all rings?

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- We intoduce the notion of an equivariant homology theory.
- We present the general formulation of the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We discuss equivariant Chern characters.
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#### Definition ( homology theory)

A *G*-homology theory  $\mathcal{H}_*$  is a covariant functor from the category of *G*-*CW*-pairs to the category of  $\mathbb{Z}$ -graded  $\Lambda$ -modules together with natural transformations

$$\partial_n(X,A) \colon \mathcal{H}_n(X,A) \to \mathcal{H}_{n-1}(A)$$

for  $n \in \mathbb{Z}$  satisfying the following axioms:

- G-homotopy invariance;
- Long exact sequence of a pair;
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$$\operatorname{ind}_{\alpha} : \mathcal{H}_{n}^{H}(X, A) \rightarrow \mathcal{H}_{n}^{G}(\operatorname{ind}_{\alpha}(X, A))$$

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- Bijectivity
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We get equivariant homology theories

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# The general formulation of the Isomorphism Conjectures

#### Conjecture (K-theoretic Farrell-Jones-Conjecture)

The K-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCVC}}(G), \mathbf{K}_R) \to H_n^G(pt, \mathbf{K}_R) = K_n(RG)$$

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• Let  $\mathcal{H}_*$  be a (non-equivariant) homology theory. There is the Atiyah-Hirzebruch spectral sequence which converges to  $\mathcal{H}_{p+q}(X)$  and has as  $E^2$ -term

$$E_{p,q}^2 = H_p(X; \mathcal{H}_q(\mathrm{pt})).$$

Rationally it collapses completely. Namely, one has the following result

#### Theorem (Non-equivariant Chern character, Dold (1962))

Let  $\mathcal{H}_*$  be a homology theory with values in  $\Lambda$ -modules for  $\mathbb{Q} \subseteq \Lambda$ . Then there exists for every  $n \in \mathbb{Z}$  and every CW-complex X a natural isomorphism

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where  $D_{p,q}$  sends  $[f: (S^{p+k}, pt) \to (S^k \wedge X_+, pt)] \otimes \eta$  to the image of  $\eta$  under the composite

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Wolfgang Lück (Münster, Germany)

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#### • We want to extend this to the equivariant setting.

- This requires an extra structure on the coefficients of an equivariant homology theory H<sup>?</sup><sub>\*</sub>.
- We define a covariant functor called induction

ind: 
$$\mathcal{FGI} \to \Lambda\text{-}Mod$$

from the category  $\mathcal{FGI}$  of finite groups with injective group homomorphisms as morphisms to the category of  $\Lambda$ -modules as follows. It sends *G* to  $\mathcal{H}_n^G(\text{pt})$  and an injection of finite groups  $\alpha: H \to G$  to the morphism given by the induction structure

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- We want to extend this to the equivariant setting.
- This requires an extra structure on the coefficients of an equivariant homology theory H<sup>?</sup><sub>\*</sub>.
- We define a covariant functor called induction

ind : 
$$\mathcal{FGI} \rightarrow \Lambda$$
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#### Definition (Mackey extension)

We say that  $\mathcal{H}^{?}_{*}$  has a Mackey extension if for every  $n \in \mathbb{Z}$  there is a contravariant functor called restriction

 $\text{res}\colon \mathcal{FGI} \to \Lambda\text{-}\text{Mod}$ 

such that these two functors ind and res agree on objects and satisfy the double coset formula ,i.e., we have for two subgroups  $H, K \subset G$  of the finite group G

$$\operatorname{res}_{G}^{K} \circ \operatorname{ind}_{H}^{G} = \sum_{KgH \in K \setminus G/H} \operatorname{ind}_{c(g):H \cap g^{-1}Kg \to K} \circ \operatorname{res}_{H}^{H \cap g^{-1}Kg}$$

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- In every case we will consider such a Mackey extension does exist and is given by an actual restriction.
- For instance for H<sup>?</sup><sub>0</sub>(−; K<sup>top</sup>) induction is the functor complex representation ring R<sub>C</sub> with respect to induction of representations. The restriction part is given by the restriction of representations.

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Let  $\mathcal{H}'_*$  be a equivariant homology theory with values in  $\Lambda$ -modules for  $\mathbb{Q} \subseteq \Lambda$ . Suppose that  $\mathcal{H}^?_*$  has a Mackey extension. Let I be the set of conjugacy classes (H) of finite subgroups H of G. Then there is for every group G, every proper G-CW-complex X and every  $n \in \mathbb{Z}$  a natural isomorphism called equivariant Chern character

$$\mathsf{ch}_n^G \colon \bigoplus_{p+q=n} \bigoplus_{(H)\in I} H_p(C_G H \setminus X^H; \Lambda) \otimes_{\Lambda[W_G H]} S_H\left(\mathcal{H}_q^H(*)\right) \xrightarrow{\cong} \mathcal{H}_n^G(X).$$

- $C_GH$  is the centralizer and  $N_GH$  the normalizer of  $H \subseteq G$ ;
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Let G be finite. Then the map

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is surjective after inverting |G|, where  $C \subset G$  runs through the cyclic subgroups of G.

Let *C* be a finite cyclic group. The Artin defect is the cokernel of the map

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# Let K<sup>G</sup><sub>\*</sub> = H<sup>?</sup><sub>\*</sub>(-; K<sup>top</sup>) be equivariant topological *K*-theory. We get for a finite subgroup H ⊆ G

$$K_n^G(G/H) = K_n^H(\text{pt}) = \begin{cases} R_{\mathbb{C}}(H) & \text{if } n \text{ is even;} \\ \{0\} & \text{if } n \text{ is odd.} \end{cases}$$

S<sub>H</sub> (K<sup>H</sup><sub>q</sub>(\*)) ⊗<sub>Z</sub> Q = 0 if H is not cyclic and q is even or if q is odd.
 S<sub>C</sub> (K<sup>C</sup><sub>q</sub>(\*)) ⊗<sub>Z</sub> Q = θ<sub>C</sub> · R<sub>C</sub>(C) ⊗<sub>Z</sub> Q if C is finite cyclic and q is

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# Corollary (Rational computation of $K_*^G(\underline{E}G)$ )

For every group G and every  $n \in \mathbb{Z}$  we obtain an isomorphism

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Equivariant homology theories

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Let G be a finite group. Then there are isomorphisms of abelian groups

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- The rational homology of each of the centralizers of elements in  $con_2(G)$  and  $con_3(G)$  agrees with the one of the trivial group.
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# $$\begin{split} & \mathcal{K}^0(\textit{BSL}_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong & \mathbb{Q} \times (\mathbb{Q}_2^{\widehat{}})^4 \times (\mathbb{Q}_3^{\widehat{}})^2; \\ & \mathcal{K}^1(\textit{BSL}_3(\mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{Q} &\cong & 0. \end{split}$$

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- Let *G* be a discrete group. Let  $\mathcal{MFIN}$  be the subset of  $\mathcal{FIN}$  consisting of elements in  $\mathcal{FIN}$  which are maximal in  $\mathcal{FIN}$ .
- Assume that *G* satisfies the following assertions:
  - (M) Every non-trivial finite subgroup of G is contained in a unique maximal finite subgroup;
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- Here are some examples of groups *G* which satisfy conditions (M) and (NM):
  - Extensions  $1 \to \mathbb{Z}^n \to G \to F \to 1$  for finite F such that the conjugation action of F on  $\mathbb{Z}^n$  is free outside  $0 \in \mathbb{Z}^n$ ;
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For such a group there is a nice model for <u>E</u>G with as few non-free cells as possible. Let {(M<sub>i</sub>) | i ∈ I} be the set of conjugacy classes of maximal finite subgroups of M<sub>i</sub> ⊆ G. By attaching free G-cells we get an inclusion of G-CW-complexes j<sub>1</sub>: ∐<sub>i∈I</sub> G ×<sub>M<sub>i</sub></sub> EM<sub>i</sub> → EG.

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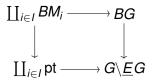
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- We begin with the case  $H \neq \{1\}$ . Because of conditions (M) and (NM) there is precisely one index  $i_0 \in I$  such that H is subconjugated to  $M_{i_0}$  and is not subconjugated to  $M_i$  for  $i \neq i_0$ . We get

$$\left(\prod_{i\in I} G/M_i\right)^H = (G/M_{i_0})^H = \text{pt.}$$

It remains to treat H = {1}. Since u<sub>1</sub> is a non-equivariant homotopy equivalence and j<sub>1</sub> is a cofibration, f<sub>1</sub> is a non-equivariant homotopy equivalence. Hence <u>E</u>G is contractible.

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The associated Mayer-Vietoris sequence yields

$$\dots \to \widetilde{H}_{p+1}(G \backslash \underline{E}G) \to \bigoplus_{i \in I} \widetilde{H}_p(BM_i) \to \widetilde{H}_p(BG)$$
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• In particular we obtain an isomorphism for  $p \ge \dim(\underline{E}G) + 2$ 

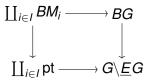
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Equivariant homology theories

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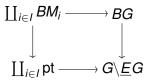
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Then there is an isomorphism

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