Some open problems about aspherical closed manifolds

Wolfgang Lück

Abstract We discuss some open and interesting problems about aspherical closed manifolds including topologically rigidity, Poincaré duality groups and L^2-invariants.

0 Introduction

This article is devoted to aspherical closed manifolds and open conjectures, problems and questions about them. All the problems stated here are very interesting and hard. Any progress towards an answer is welcome and valuable. We hope that a reader may be motivated by this note to study them.

We will address the questions whether an aspherical closed manifold is topologically rigid, whether a finitely presented Poincaré duality group is the fundamental group of an aspherical closed manifold, whether an aspherical closed manifold carries an S^1-action or a Riemannian metric with positive scalar curvature, and finally state some conjectures about the possible values of L^2-Betti numbers and L^2-torsion of the universal covering of an aspherical closed manifold and the homological growth in a tower of finite coverings.

1 Basics about aspherical CW-complexes

A CW-complex X is called *aspherical* if it is connected and the nth homotopy group $\pi_n(X)$ vanish for $n \geq 2$, or, equivalently, it is connected and its universal covering is contractible. Two aspherical CW-complexes are homotopy equivalent if and only if their fundamental groups are isomorphic. This follows from the fact

Wolfgang Lück
Mathematical Institut of the University at Bonn, Endenicher Allee 60, 53115 Bonn, Germany
e-mail: wolfgang.lueck@him.uni-bonn.de

that for any connected CW-complex X and any aspherical CW-complex Y two maps $f_0, f_1 \colon \pi_1(X) \to \pi_1(Y)$ are homotopic if and only if for one (and hence all points) $x \in X$ there exists path w from $f_0(x)$ to $f_1(x)$ such that the composite of the obvious map $c_w \colon \pi_1(X, f_0(x)) \to \pi_1(Y, f_1(x))$ given by conjugation with w and $\pi_1(f_0,x) \colon \pi_1(X,x) \to \pi_1(Y, f_0(x))$ is $\pi_1(f_1,x) \colon \pi_1(X,x) \to \pi_1(Y, f_1(x))$. So the homotopy theory of aspherical CW-complexes is completely determined by their fundamental groups.

Given any group G, there exists a connected aspherical CW-complex X with $\pi_1(X) \cong G$. Since X is unique up to homotopy, one often denotes such a space by BG or $K(G,1)$. One defines the homology $H_*(G)$ of a group G by $H_*(BG)$ and this definition is independent of the choice of model BG by homotopy invariance.

2 Basics about aspherical closed manifolds

We are interested in aspherical closed (topological or smooth) manifolds. These exists in abundance.

2.1 Non-positive curvature

Let M be a closed smooth manifold. Suppose that it possesses a Riemannian metric whose sectional curvature is non-positive. Then the universal covering $\widetilde{M}$ inherits a complete Riemannian metric whose sectional curvature is non-positive. The Hadamard-Cartan Theorem (see [31, 3.87 on page 134]) implies that $\widetilde{M}$ is diffeomorphic to $\mathbb{R}^n$. Hence M is aspherical.

2.2 Low-dimensions

A connected closed 1-dimensional manifold is homeomorphic to S^1 and hence aspherical.

Let M be a connected closed 2-dimensional manifold. Then M is either aspherical or homeomorphic to S^2 or $\mathbb{RP}^2$. The following statements are equivalent: i.) M is aspherical. ii.) M admits a Riemannian metric which is *flat*, i.e., with sectional curvature constant 0, or which is *hyperbolic*, i.e., with sectional curvature constant -1. iii) The universal covering of M is homeomorphic to $\mathbb{R}^2$.

A connected closed 3-manifold M is called *prime* if for any decomposition as a connected sum $M \cong M_0 \sharp M_1$ one of the summands M_0 or M_1 is homeomorphic to S^3. It is called *irreducible* if any embedded sphere S^2 bounds an embedded disk D^3. Every irreducible closed 3-manifold is prime. A prime closed 3-manifold is either irreducible or an S^2-bundle over S^1 (see [37, Lemma 3.13 on page 28]). A closed orientable 3-manifold is aspherical if and only if it is irreducible and has infinite fundamental group. This follows from the Sphere Theorem [37, Theorem 4.3 on page 40].

2.3 Torsionfree discrete subgroups of almost connected Lie groups

Let L be a Lie group with finitely many path components. Let $K \subseteq L$ be a maximal compact subgroup. Let $G \subseteq L$ be a discrete torsionfree subgroup. Then $M = G\backslash L/K$ is an aspherical closed manifold with fundamental group G since its universal covering L/K is diffeomorphic to $\mathbb{R}^n$ for appropriate n (see [36, Theorem 1. in Chapter VI]). Examples for M are hyperbolic manifolds.

2.4 Hyperbolization

A very important construction of aspherical manifolds comes from the *hyperbolization technique* due to Gromov [33]. It turns a cell complex into a non-positively curved (and hence aspherical) polyhedron. The rough idea is to define this procedure for simplices such that it is natural under inclusions of simplices and then define the hyperbolization of a simplicial complex by gluing the results for the simplices together as described by the combinatorics of the simplicial complex. The goal is to achieve that the result shares some of the properties of the simplicial complexes one has started with, but additionally to produce a non-positively curved and hence aspherical polyhedron. Since this construction preserves local structures, it turns manifolds into manifolds.

We briefly explain what the *orientable hyperbolization procedure* gives. Further expositions of this construction can be found in [16, 20, 21, 22]. We start with a finite-dimensional simplicial complex Σ and assign to it a cubical cell complex $h(\Sigma)$ and a natural map $c\colon h(\Sigma) \to \Sigma$ with the following properties:

1. $h(\Sigma)$ is non-positively curved and in particular aspherical;
2. The natural map $c\colon h(\Sigma) \to \Sigma$ induces a surjection on the integral homology;
3. $\pi_1(f)\colon \pi_1(h(\Sigma)) \to \pi_1(\Sigma)$ is surjective;
4. If Σ is an oriented closed manifold, then
    a. $h(\Sigma)$ is an oriented closed manifold;
    b. The natural map $c\colon h(\Sigma) \to \Sigma$ has degree one;
    c. There is a stable isomorphism between the tangent bundle $Th(\Sigma)$ and the pullback $c^*T\Sigma$;

One can deduce from this construction that the condition aspherical does not impose any restrictions on the characteristic numbers of a manifold or on its bordism class, see [20, Remarks 15.1] and [22, Theorem B]. Moreover, it can be used to construct aspherical closed manifolds with rather exotics properties, for instance examples which do not possess a triangulation, whose universal covering is not homeomorphic to $\mathbb{R}^n$, whose fundamental group contains an infinite divisible abelian group or has an unsolvable word problem. For such exotic examples and more information about aspherical closed manifolds we refer for instance to [9, 20, 22, 47].

3 The Borel Conjecture

In this section we deal with

Conjecture 1 (Borel Conjecture for a group G). If M and N are aspherical closed manifolds with $\pi_1(M) \cong \pi_1(N) \cong G$, then M and N are homeomorphic and any homotopy equivalence $M \to N$ is homotopic to a homeomorphism.

The main tool to attack the Borel Conjecture is surgery theory and the Farrell-Jones Conjecture. We consider the following special version of the Farrell-Jones Conjecture.

Conjecture 2 (Farrell-Jones Conjecture for torsionfree groups and integer coefficients). Let G be a torsionfree group Then:

1. $K_n(\mathbb{Z}G) = 0$ for $n \leq -1$;
2. The reduced projective class group $\widetilde{K}_0(\mathbb{Z}G)$ vanishes;
3. The Whitehead group $\mathrm{Wh}(G)$ vanishes;
4. For any homomorphism $w\colon G \to \{\pm 1\}$ the w-twisted L-theoretic assembly map $H_n(BG; {}^w\mathbf{L}^{\langle -\infty \rangle}) \xrightarrow{\cong} L_n^{\langle -\infty \rangle}(RG, w)$ is bijective.

The relevance of the Conjecture 2 for the Borel Conjecture comes from the next theorem whose proof is based on surgery theory.

Theorem 1 (The Farrell-Jones Conjecture and the Borel Conjecture). *Suppose that G satisfies the version of the Farrell-Jones Conjecture stated in Conjecture 2.*

Then the Borel Conjecture is true for aspherical closed manifolds of dimension ≥ 5 *with G as fundamental group. It is true for aspherical closed manifolds of dimension* 4 *with G as fundamental group if G is good in the sense of Freedman (see [29], [30]).*

Remark 1 (The Borel Conjecture in low dimensions). The Borel Conjecture is true in dimension ≤ 2 by the classification of closed manifolds of dimension ≤ 2. It is true in dimension 3 if Thurston's Geometrization Conjecture is true. This follows from results of Waldhausen (see Hempel [37, Lemma 10.1 and Corollary 13.7]) and Turaev (see [61]) as explained for instance in [42, Section 5]. A proof of Thurston's Geometrization Conjecture is given in [50] following ideas of Perelman.

Remark 2 (The Borel Conjecture does not hold in the smooth category). The Borel Conjecture 1 is false in the smooth category, i.e., if one replaces topological manifold by smooth manifold and homeomorphism by diffeomorphism. The torus T^n for $n \geq 5$ is an example (see [62, 15A]). Other counterexample involving negatively curved manifolds are constructed by Farrell-Jones [24, Theorem 0.1].

Remark 3 (The Borel Conjecture versus Mostow rigidity). A version of *Mostow rigidity* says for two closed hyperbolic manifolds N_0 and N_1 that they are isometrically diffeomorphic if and only if $\pi_1(N_0) \cong \pi_1(N_1)$ and any homotopy equivalence $N_0 \to N_1$ is homotopic to an isometric diffeomorphism.

One may view the Borel Conjecture as the topological version of Mostow rigidity. The conclusion in the Borel Conjecture is weaker, one gets only homeomorphisms and not isometric diffeomorphisms, but the assumption is also weaker, since there are many more aspherical closed topological manifolds than hyperbolic closed manifolds.

The following is known about the Farrell-Jones Conjecture, see for instance [3, 4, 5, 6, 7, 39, 59, 63].

Theorem 2. *Let $\mathscr{C}$ be the smallest class of groups satisfying:*

- *Every hyperbolic group belongs to $\mathscr{C}$;*
- *Every group that acts properly, isometrically and cocompactly on a complete proper* $\operatorname{CAT}(0)$*-space belongs to $\mathscr{C}$;*
- *Every lattice in an almost connected Lie group belongs to $\mathscr{C}$;*
- *Every virtually solvable group belongs to $\mathscr{C}$;*
- *Every arithmetic groups belongs to $\mathscr{C}$;*
- *The fundamental group of any* 3*-manifold (possibly with boundary and possibly non-compact) belongs to $\mathscr{C}$;*
- *If G_1 and G_2 belong to $\mathscr{C}$, then both $G_1 * G_2$ and $G_1 \times G_2$ belong to $\mathscr{C}$;*
- *If H is a subgroup of G and $G \in \mathscr{C}$, then $H \in \mathscr{C}$;*
- *Let $\{G_i \mid i \in I\}$ be a directed system of groups (with not necessarily injective structure maps) such that $G_i \in \mathscr{C}$ for every $i \in I$. Then the directed colimit* $\operatorname{colim}_{i \in I} G_i$ *belongs to $\mathscr{C}$.*

Then every group G in $\mathscr{C}$ satisfies the K- and L-theoretic Farrell-Jones Conjecture with coefficients in additive categories and with finite wreath products, and in particular the version of the Farrell-Jones Conjecture stated in Conjecture 2.

For more information about the Borel and the Farrell-Jones Conjecture and literature about them we refer for instance to [25, 46, 49].

4 Poincaré duality groups

The following definition is due to Johnson-Wall [38].

Definition 1 (Poincaré duality group).
A group G is called a *Poincaré duality group of dimension n* if the following conditions holds:

1. The group G is of type FP, i.e., the trivial $\mathbb{Z}G$-module $\mathbb{Z}$ possesses a finite-dimensional projective $\mathbb{Z}G$-resolution by finitely generated projective $\mathbb{Z}G$-modules;
2. We get an isomorphism of abelian groups

$$H^i(G;\mathbb{Z}G) \cong \begin{cases} \{0\} & \text{for } i \neq n; \\ \mathbb{Z} & \text{for } i = n. \end{cases}$$

If G is the fundamental group of an aspherical closed manifold of dimension n, then it is finitely presented and a Poincaré duality group of dimension n. This leads to

Conjecture 3 (Poincaré duality groups). A finitely presented group is a n-dimensional Poincaré duality group if and only if it is the fundamental group of an aspherical closed n-dimensional topological manifold.

Conjecture 3 is known to be true if $n = 1,2$. This is obvious for $n = 1$ and for $n = 2$ proved in [23, Theorem 2].

A topological space X is called an *absolute neighborhood retract* or briefly ANR if for every normal space Z, every closed subset $Y \subseteq Z$ and every (continuous) map $f\colon Y \to X$ there exists an open neighborhood U of Y in Z together with an extension $F\colon U \to Z$ of f to U. A *compact n-dimensional homology* ANR-*manifold* X is a compact absolute neighborhood retract such that it has a countable basis for its topology, has finite topological dimension and for every $x \in X$ the abelian group $H_i(X, X - \{x\})$ is trivial for $i \neq n$ and infinite cyclic for $i = n$. A closed n-dimensional topological manifold is an example of a compact n-dimensional homology ANR-manifold (see [19, Corollary 1A in V.26 page 191]).

The *disjoint disk property* says that for any $\varepsilon > 0$ and maps $f, g\colon D^2 \to M$ there are maps $f', g'\colon D^2 \to M$ so that the distance between f and f' and the distance between g and g' are bounded by ε and $f'(D^2) \cap g'(D^2) = \emptyset$.

Theorem 3. *Let G be a finitely presented group and $n \geq 6$ be a natural number. Suppose that G satisfies the version of the Farrell-Jones Conjecture 2.*

Then G is the fundamental group of a compact homology ANR-*manifold of dimension n satisfying the disjoint disk property if and only if G is an n-dimensional Poincaré duality group.*

Proof. See [13, Main Theorem on page 439 and Section 8], [14, Theorem A and Theorem B], and [57, Remark 25.13 on page 297].

One would prefer if in the conclusion of Theorem 3 one could replace "compact homology ANR-manifold" by "closed topological manifold". The remaining obstruction is the *resolution obstruction* of Quinn which takes values in $1 + 8 \cdot \mathbb{Z}$. Any element in $1 + 8 \cdot \mathbb{Z}$ can be realized by an appropriate compact homology

ANR-manifold as its *resolution obstruction*. There are compact homology ANR-manifolds that are not homotopy equivalent to closed manifolds. But no example of an aspherical compact homology ANR-manifold that is not homotopy equivalent to a closed topological manifold is known. So we could replace in the conclusion of Theorem 3 "compact homology ANR-manifold" by "closed topological manifold" if the following question has a positive answer.

Question 1 (Vanishing of the resolution obstruction in the aspherical case). Is every aspherical compact homology ANR-manifold having the DDP homotopy equivalent to a closed manifold?

We refer for instance to [13, 26, 55, 56, 57] for more information about this topic.

The question which hyperbolic groups arise as fundamental groups of aspherical closed manifolds of dimension n and which torsionfree hyperbolic groups have a sphere S^{n-1} as boundary is answered by Bartels-Lück-Weinberger [8] in dimension $n \geq 6$.

5 S^1-actions

Let M be a closed aspherical manifold with a non-trivial S^1-action. Then the S^1-action has only finite isotropy groups, the inclusion of any orbit induces an injection on the fundamental group and the center of $\pi_1(X)$ contains an infinite normal cyclic subgroup. A proof can be found for instance in [17] or [44, Corollary 1.43 on page 48]. Conner-Raymond [17] conjectured that an aspherical closed manifold whose fundamental group has a non-trivial center admits a non-trivial S^1-action. This conjecture has been disproved Cappell-Weinberger-Yan [15]. One may still ask the following question

Question 2 (S^1-actions). If M is an aspherical closed manifold whose fundamental group has a non-trivial center, is there a finite covering which admits a non-trivial S^1-action?

6 Fiber bundles

Question 3 (Fiber bundles). Let $f\colon M \to N$ be a map of aspherical closed manifolds which induces a surjection on fundamental groups.

Under which conditions is it homotopy equivalent to the projection of a locally trivial topological fiber bundle (or to a Manifold Approximate Fibration)?

A necessary condition for a positive answer is that the homotopy fiber has the homotopy type of a finite CW-complex. If the homotopy fiber is a point, or equivalently, if f is a homotopy equivalence, a positive answer (for a locally trivial fiber bundle) is equivalent to the statement that f is homotopic to a homeomorphism, in other words Question 3 becomes the Borel Conjecture 1.

7 L^2-invariants

Next we mention some prominent conjectures about aspherical closed manifolds and L^2-invariants. For more information about these conjectures and their status we refer to [10, 44, 45]. We denote by $b_p^{(2)}(\widetilde{M})$ the p-th L^2-Betti number and by $\rho^{(2)}(\widetilde{M})$ the L^2-torsion of the universal covering $\widetilde{M}$ of a closed manifold M.

7.1 The Hopf and the Singer Conjecture

Conjecture 4 (Hopf Conjecture). If M is an aspherical closed manifold of even dimension, then

$$(-1)^{\dim(M)/2} \cdot \chi(M) \geq 0.$$

If M is a closed Riemannian manifold of even dimension with sectional curvature $\sec(M)$, then

$$\begin{array}{rl} (-1)^{\dim(M)/2} \cdot \chi(M) > 0 & \text{if } \sec(M) < 0; \\ (-1)^{\dim(M)/2} \cdot \chi(M) \geq 0 & \text{if } \sec(M) \leq 0; \\ \chi(M) \geq 0 & \text{if } \sec(M) \geq 0; \\ \chi(M) > 0 & \text{if } \sec(M) > 0. \end{array}$$

Conjecture 5 (Singer Conjecture). If M is an aspherical closed manifold, then

$$b_p^{(2)}(\widetilde{M}) = 0 \qquad \text{if } 2p \neq \dim(M).$$

If M is a closed connected Riemannian manifold with negative sectional curvature, then

$$b_p^{(2)}(\widetilde{M}) \begin{cases} = 0 & \text{if } 2p \neq \dim(M); \\ > 0 & \text{if } 2p = \dim(M). \end{cases}$$

7.2 L^2-torsion and aspherical closed manifolds

Conjecture 6 (L^2-torsion for aspherical closed manifolds). If M is an aspherical closed manifold of odd dimension, then $\widetilde{M}$ is det-L^2-acyclic and

$$(-1)^{\frac{\dim(M)-1}{2}} \cdot \rho^{(2)}(\widetilde{M}) \geq 0.$$

If M is a closed connected Riemannian manifold of odd dimension with negative sectional curvature, then $\widetilde{M}$ is det-L^2-acyclic and

$$(-1)^{\frac{\dim(M)-1}{2}} \cdot \rho^{(2)}(\widetilde{M}) > 0.$$

If M is an aspherical closed manifold whose fundamental group contains an amenable infinite normal subgroup, then $\widetilde{M}$ is det-L^2-acyclic and

$$\rho^{(2)}(\widetilde{M}) = 0.$$

7.3 Simplicial volume and L^2-invariants

Conjecture 7 (Simplicial volume and L^2-invariants). Let M be an aspherical closed orientable manifold. Suppose that its simplicial volume $||M||$ vanishes. Then $\widetilde{M}$ is of determinant class and

$$\begin{aligned} b_p^{(2)}(\widetilde{M}) &= 0 \qquad \text{for } p \geq 0; \\ \rho^{(2)}(\widetilde{M}) &= 0. \end{aligned}$$

7.4 Zero-in-the-Spectrum Conjecture

Conjecture 8 (Zero-in-the-spectrum Conjecture). Let $\widetilde{M}$ be a complete Riemannian manifold. Suppose that $\widetilde{M}$ is the universal covering of an aspherical closed Riemannian manifold M (with the Riemannian metric coming from M). Then for some $p \geq 0$ zero is in the Spectrum of the minimal closure

$$(\Delta_p)_{\min}\colon \operatorname{dom}\bigl((\Delta_p)_{\min}\bigr) \subset L^2\Omega^p(\widetilde{M}) \to L^2\Omega^p(\widetilde{M})$$

of the Laplacian acting on smooth p-forms on $\widetilde{M}$.

7.5 Homological growth

Here is a generalization of a conjecture due to Bergeron-Venkatesh [10, Conjecture 1.3].

Conjecture 9 (Homological growth and L^2-torsion for aspherical closed manifolds).

Let M be an aspherical closed manifold of dimension d and fundamental group $G = \pi_1(M)$. Let $G = G_0 \supseteq G_1 \supseteq \cdots$ be a descending sequence of in G normal subgroups $[G : G_i]$ with trivial intersection $\bigcap_{i \geq 0} G_i = \{1\}$. Put $M[i] = G_i \backslash \widetilde{M}$, where $\widetilde{M}$ is the universal covering. Let F be an field. Then

1. We get for any $p \geq 0$
$$b_p^{(2)}(\widetilde{M}) = \lim_{i \to \infty} \frac{b_n(M[i]; F)}{[G : G_i]};$$

2. We get for any natural number p with $2p+1 \neq d$

$$\lim_{i\to\infty} \frac{\ln\big(\big|\mathrm{tors}\big(H_p(M[i])\big)\big|\big)}{[G:G_i]} = 0,$$

and we get in the case $d = 2p+1$

$$\lim_{i\to\infty} \frac{\ln\big(\big|\mathrm{tors}\big(H_n(M[i])\big)\big|\big)}{[G:G_i]} = (-1)^p \cdot \rho^{(2)}\big(\widetilde{M}\big).$$

Some evidence for Conjecture 9 comes from [10] and [48].

8 Positive scalar curvature

Conjecture 10. An aspherical closed smooth manifold does not admit a Riemannian metric of positive scalar curvature.

Some evidence comes from the following fact. Let M be an aspherical closed smooth manifold whose fundamental group $\pi = \pi_1(M)$ satisfies the *strong Novikov Conjecture*, i.e., the assembly map $K_n(B\pi) \to K_n(C_r^*(\pi))$ from the K-homology of BG to the topological K-theory of the reduced group C^*-algebra is rationally injective for all $n \in \mathbb{Z}$. Then M carries no Riemannian metric of positive scalar curvature, see [58, Theorem 3.5]. Moreover, M satisfies the Zero-in-the-Spectrum Conjecture 8, see [43, Corollary 4]. We refer to [49, Section 5.1.3] for a discussion about the large class of groups for which the assembly map $K_n(BG) \to K_n(C_r^*(G))$ is known to be injective or rationally injective. More information about the Novikov Conjecture can be found in for instance in [27, 28, 41].

9 Random closed manifolds

The idea of a random group has successfully been used to construct groups with certain properties, see for instance [2, 32, 34, 35, 51, 52, 53, 60, 64]. For example, in a precise statistical sense almost all finitely presented groups are torsionfree hyperbolic and in particular have a finite model for their classifying space.

It is not clear what it means in a precise sense to talk about a random closed manifold. Nevertheless, the author's intuition is that almost all closed manifolds are aspherical. (A related question would be whether a random closed smooth manifold admits a Riemannian metric with non-positive sectional curvature.) It is certainly true in dimension 2 since only finitely many closed surfaces are not aspherical. The characterization of closed 3-dimensional manifolds in Subsection 2.2 seems to fit as well.

A closed manifold M is called *asymmetric* if every finite group which acts effectively on M is trivial. This is equivalent to the statement that for any choice of Riemannian metric on M the group of isometries is trivial (see [40, Introduction]). A survey on asymmetric closed manifolds can be found in [54]. The first constructions of asymmetric aspherical closed manifolds are due to Connor-Raymond-Weinberger [18]. The first simply-connected asymmetric manifold has been constructed by Kreck [40] answering a question of Raymond and Schultz [12, page 260] which was repeated by Adem and Davis [1] in their problem list. Raymond and Schultz expressed also their feeling that a random manifold should be asymmetric. Borel has shown that an aspherical closed manifold is asymmetric if its fundamental group is centerless and its outer automorphism group is torsionfree (see the manuscript "On periodic maps of certain $K(\pi,1)$" in [11, pages 57–60]).

This leads to the intuitive assertion:

Almost all closed manifolds are aspherical, topologically rigid in the sense of the Borel Conjecture 1 and asymmetric.

References

1. A. Adem and J. F. Davis. Topics in transformation groups. In *Handbook of geometric topology*, pages 1–54. North-Holland, Amsterdam, 2002.
2. G. Arzhantseva and T. Delzant. Examples of random groups. Preprint, 2008.
3. A. Bartels, S. Echterhoff, and W. Lück. Inheritance of isomorphism conjectures under colimits. In Cortinaz, Cuntz, Karoubi, Nest, and Weibel, editors, *K-Theory and noncommutative geometry*, EMS-Series of Congress Reports, pages 41–70. European Mathematical Society, 2008.
4. A. Bartels, F. T. Farrell, and W. Lück. The Farrell-Jones Conjecture for cocompact lattices in virtually connected Lie groups. *J. Amer. Math. Soc.*, 27(2):339–388, 2014.
5. A. Bartels and W. Lück. The Borel conjecture for hyperbolic and CAT(0)-groups. *Ann. of Math. (2)*, 175:631–689, 2012.
6. A. Bartels, W. Lück, and H. Reich. The K-theoretic Farrell-Jones conjecture for hyperbolic groups. *Invent. Math.*, 172(1):29–70, 2008.
7. A. Bartels, W. Lück, H. Reich, and H. Rüping. K- and L-theory of group rings over $\mathrm{GL}_n(\mathbf{Z})$. *Publ. Math., Inst. Hautes Étud. Sci.*, 119:97–125, 2014.
8. A. Bartels, W. Lück, and S. Weinberger. On hyperbolic groups with spheres as boundary. *Journal of Differential Geometry*, 86(1):1–16, 2010.
9. I. Belegradek. Aspherical manifolds, relative hyperbolicity, simplicial volume and assembly maps. *Algebr. Geom. Topol.*, 6:1341–1354 (electronic), 2006.
10. N. Bergeron and A. Venkatesh. The asymptotic growth of torsion homology for arithmetic groups. *J. Inst. Math. Jussieu*, 12(2):391–447, 2013.
11. A. Borel. *Œuvres: collected papers. Vol. III, 1969–1982*. Springer-Verlag, Berlin, 1983.
12. W. Browder and W. C. Hsiang. Some problems on homotopy theory manifolds and transformation groups. In *Algebraic and geometric topology (Proc. Sympos. Pure Math., Stanford Univ., Stanford, Calif., 1976), Part 2*, Proc. Sympos. Pure Math., XXXII, pages 251–267. Amer. Math. Soc., Providence, R.I., 1978.
13. J. Bryant, S. Ferry, W. Mio, and S. Weinberger. Topology of homology manifolds. *Ann. of Math. (2)*, 143(3):435–467, 1996.
14. J. Bryant, S. Ferry, W. Mio, and S. Weinberger. Desingularizing homology manifolds. *Geom. Topol.*, 11:1289–1314, 2007.

15. S. E. Cappell, S. Weinberger, and M. Yan. Closed aspherical manifolds with center. *Journal of Topology*, 6:1009–1018, 2014.
16. R. M. Charney and M. W. Davis. Strict hyperbolization. *Topology*, 34(2):329–350, 1995.
17. P. E. Conner and F. Raymond. Actions of compact Lie groups on aspherical manifolds. In *Topology of Manifolds (Proc. Inst., Univ. of Georgia, Athens, Ga., 1969)*, pages 227–264. Markham, Chicago, Ill., 1970.
18. P. E. Conner, F. Raymond, and P. J. Weinberger. Manifolds with no periodic maps. In *Proceedings of the Second Conference on Compact Transformation Groups (Univ. Massachusetts, Amherst, Mass., 1971), Part II*, pages 81–108. Lecture Notes in Math., Vol. 299, Berlin, 1972. Springer.
19. R. J. Daverman. *Decompositions of manifolds*, volume 124 of *Pure and Applied Mathematics*. Academic Press Inc., Orlando, FL, 1986.
20. M. Davis. Exotic aspherical manifolds. In T. Farrell, L. Göttsche, and W. Lück, editors, *High dimensional manifold theory*, number 9 in ICTP Lecture Notes, pages 371–404. Abdus Salam International Centre for Theoretical Physics, Trieste, 2002. Proceedings of the summer school "High dimensional manifold theory" in Trieste May/June 2001, Number 2. http://www.ictp.trieste.it/~pub_off/lectures/vol9.html.
21. M. W. Davis. *The geometry and topology of Coxeter groups*, volume 32 of *London Mathematical Society Monographs Series*. Princeton University Press, Princeton, NJ, 2008.
22. M. W. Davis and T. Januszkiewicz. Hyperbolization of polyhedra. *J. Differential Geom.*, 34(2):347–388, 1991.
23. B. Eckmann and P. A. Linnell. Poincaré duality groups of dimension two. II. *Comment. Math. Helv.*, 58(1):111–114, 1983.
24. F. T. Farrell and L. E. Jones. Negatively curved manifolds with exotic smooth structures. *J. Amer. Math. Soc.*, 2(4):899–908, 1989.
25. F. T. Farrell and L. E. Jones. Isomorphism conjectures in algebraic K-theory. *J. Amer. Math. Soc.*, 6(2):249–297, 1993.
26. S. C. Ferry and E. K. Pedersen. Epsilon surgery theory. In *Novikov conjectures, index theorems and rigidity, Vol. 2 (Oberwolfach, 1993)*, pages 167–226. Cambridge Univ. Press, Cambridge, 1995.
27. S. C. Ferry, A. A. Ranicki, and J. Rosenberg, editors. *Novikov conjectures, index theorems and rigidity. Vol. 1*. Cambridge University Press, Cambridge, 1995. Including papers from the conference held at the Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, September 6–10, 1993.
28. S. C. Ferry, A. A. Ranicki, and J. Rosenberg, editors. *Novikov conjectures, index theorems and rigidity. Vol. 2*. Cambridge University Press, Cambridge, 1995. Including papers from the conference held at the Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, September 6–10, 1993.
29. M. H. Freedman. The topology of four-dimensional manifolds. *J. Differential Geom.*, 17(3):357–453, 1982.
30. M. H. Freedman. The disk theorem for four-dimensional manifolds. In *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Warsaw, 1983)*, pages 647–663, Warsaw, 1984. PWN.
31. S. Gallot, D. Hulin, and J. Lafontaine. *Riemannian geometry*. Springer-Verlag, Berlin, 1987.
32. É. Ghys. Groupes aléatoires (d'après Misha Gromov,…). *Astérisque*, 294:viii, 173–204, 2004.
33. M. Gromov. Hyperbolic groups. In *Essays in group theory*, pages 75–263. Springer-Verlag, New York, 1987.
34. M. Gromov. Asymptotic invariants of infinite groups. In *Geometric group theory, Vol. 2 (Sussex, 1991)*, pages 1–295. Cambridge Univ. Press, Cambridge, 1993.
35. M. Gromov. Random walk in random groups. *Geom. Funct. Anal.*, 13(1):73–146, 2003.
36. S. Helgason. *Differential geometry, Lie groups, and symmetric spaces*. American Mathematical Society, Providence, RI, 2001. Corrected reprint of the 1978 original.
37. J. Hempel. 3-*Manifolds*. Princeton University Press, Princeton, N. J., 1976. Ann. of Math. Studies, No. 86.

38. F. E. A. Johnson and C. T. C. Wall. On groups satisfying Poincaré duality. *Ann. of Math. (2)*, 96:592–598, 1972.
39. H. Kammeyer, W. Lück, and H. Rüping. The Farrell-Jones Conjecture for arbitrary lattices in virtually connected Lie groups. Preprint, arXiv:1401.0876 [math.KT], 2014.
40. M. Kreck. Simply connected asymmetric manifolds. *J. Topol.*, 2(2):249–261, 2009.
41. M. Kreck and W. Lück. *The Novikov conjecture: Geometry and algebra*, volume 33 of *Oberwolfach Seminars*. Birkhäuser Verlag, Basel, 2005.
42. M. Kreck and W. Lück. Topological rigidity for non-aspherical manifolds. *Pure and Applied Mathematics Quarterly*, 5 (3):873–914, 2009. special issue in honor of Friedrich Hirzebruch.
43. J. Lott. The zero-in-the-spectrum question. *Enseign. Math. (2)*, 42(3-4):341–376, 1996.
44. W. Lück. *L^2-Invariants: Theory and Applications to Geometry and K-Theory*, volume 44 of *Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics]*. Springer-Verlag, Berlin, 2002.
45. W. Lück. L^2-invariants from the algebraic point of view. In *Geometric and cohomological methods in group theory*, volume 358 of *London Math. Soc. Lecture Note Ser.*, pages 63–161. Cambridge Univ. Press, Cambridge, 2009.
46. W. Lück. *K*- and *L*-theory of group rings. In *Proceedings of the International Congress of Mathematicians. Volume II*, pages 1071–1098, New Delhi, 2010. Hindustan Book Agency.
47. W. Lück. Aspherical manifolds. *Bulletin of the Manifold Atlas 2012*, pages 1–17, 2012.
48. W. Lück. Approximating L^2-invariants and homology growth. *Geom. Funct. Anal.*, 23(2):622–663, 2013.
49. W. Lück and H. Reich. The Baum-Connes and the Farrell-Jones conjectures in *K*- and *L*-theory. In *Handbook of K-theory. Vol. 1, 2*, pages 703–842. Springer, Berlin, 2005.
50. J. Morgan and G. Tian. Completion of the proof of the Geometrization Conjecture. Preprint, arXiv:0809.4040v1 [math.DG], 2008.
51. Y. Ollivier. *A January 2005 invitation to random groups*, volume 10 of *Ensaios Matemáticos [Mathematical Surveys]*. Sociedade Brasileira de Matemática, Rio de Janeiro, 2005.
52. A. Y. Olshanskii. Almost every group is hyperbolic. *Internat. J. Algebra Comput.*, 2(1):1–17, 1992.
53. P. Pansu. Groupes aléatoires. In *Groupes et géométrie*, volume 2003 of *SMF Journ. Annu.*, pages 37–49. Soc. Math. France, Paris, 2003.
54. V. Puppe. Do manifolds have little symmetry? *J. Fixed Point Theory Appl.*, 2(1):85–96, 2007.
55. F. Quinn. Resolutions of homology manifolds and the topological characterization of manifolds. *Inventiones Mathematicae*, 72:267–284, 1983.
56. F. Quinn. An obstruction to the resolution of homology manifolds. *Michigan Math. J.*, 34(2):285–291, 1987.
57. A. A. Ranicki. *Algebraic L-theory and topological manifolds*. Cambridge University Press, Cambridge, 1992.
58. J. Rosenberg. C^*-algebras, positive scalar curvature, and the Novikov conjecture. *Inst. Hautes Études Sci. Publ. Math.*, 58:197–212 (1984), 1983.
59. H. Rüping. The Farrell-Jones conjecture for S-arithmetic groups. Preprint, arXiv:1309.7236 [math.KT], 2013.
60. L. Silberman. Addendum to: "Random walk in random groups" [Geom. Funct. Anal. **13** (2003), no. 1, 73–146; mr1978492] by M. Gromov. *Geom. Funct. Anal.*, 13(1):147–177, 2003.
61. V. G. Turaev. Homeomorphisms of geometric three-dimensional manifolds. *Mat. Zametki*, 43(4):533–542, 575, 1988. translation in Math. Notes 43 (1988), no. 3-4, 307–312.
62. C. T. C. Wall. *Surgery on compact manifolds*, volume 69 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, second edition, 1999. Edited and with a foreword by A. A. Ranicki.
63. C. Wegner. The Farrell-Jones conjecture for virtually solvable groups. Preprint, arXiv:1308.2432 [math.GT], 2013.
64. A. Żuk. Property (T) and Kazhdan constants for discrete groups. *Geom. Funct. Anal.*, 13(3):643–670, 2003.