Summary, status and outlook (Lecture VI)

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Flashback

- We have formulated the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
- We have already discussed applications.
- Cliffhanger

Question (Status)

For which groups are the Farrell-Jones Conjecture and the Baum-Connes Conjecture known to be true? What are open interesting cases?

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- We briefly review the Farrell-Jones and the Baum-Connes Conjecture.
- We review applications of the Farrell-Jones and the Baum-Connes Conjecture.
- We mention other versions of the Isomorphism Conjectures.
- We explain relations between the Farrell-Jones and the Baum-Connes Conjecture.
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Review of the Isomorphism Conjectures

Conjecture (*K*-theoretic Farrell-Jones-Conjecture)

The K-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

$$H_n^G(E_{\mathcal{VCYC}}(G), \mathbf{K}_R) \to H_n^G(pt, \mathbf{K}_R) = K_n(RG)$$

is bijective for all $n \in \mathbb{Z}$.

Conjecture (*L*-theoretic Farrell-Jones-Conjecture)

The L-theoretic Farrell-Jones Conjecture with coefficients in R for the group G predicts that the assembly map

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Conjecture (Baum-Connes Conjecture)

The Baum-Connes Conjecture predicts that the assembly map

$$K_n^G(\underline{E}G) = H_n^G(E_{\mathcal{FIN}}(G), \mathbf{K}^{\text{top}}) \to H_n^G(pt, \mathbf{K}^{\text{top}}) = K_n(C_r^*(G))$$

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Review of Applications

- The following results or conjectures are consequences of the Farrell-Jones or Baum-Connes Conjecture.
- $\mathcal{FJ}_K(R)$, $\mathcal{FJ}_L(R)$ or $\mathcal{BC}$ respectively are the classes of groups which satisfy the Farrell-Jones Conjecture for K- or L-theory with coefficients in R or the Baum-Connes Conjecture respectively.

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Theorem ($K_n(\mathbb{Z}G)$ for $n \leq 1$ and torsionfree G)

We get for a torsionfree group $G \in \mathcal{FJ}(\mathbb{Z})$:

- $K_n(\mathbb{Z}G) = 0$ *for* $n \leq -1$;
- $\widetilde{K}_0(\mathbb{Z}G) = 0$;
- $\mathrm{Wh}(G) = 0$;
- *Every finitely dominated CW-complex X with $G = \pi_1(X)$ is homotopy equivalent to a finite CW-complex;*
- *Every compact h-cobordism $W = (W; M_0, M_1)$ of dimension ≥ 6 with $\pi_1(W) \cong G$ is trivial.*

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Conjecture (Kaplansky Conjecture)

The Kaplansky Conjecture says for a torsionfree group G and an integral domain R that 0 and 1 are the only idempotents in RG.

Theorem (The Farrell-Jones Conjecture and the Kaplansky Conjecture)

If F is a field of characteristic zero and the torsionfree group G belongs to $\mathcal{FJ}_K(F)$, then G and F satisfy the Kaplansky Conjecture.

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Conjecture (Borel Conjecture)

The Borel Conjecture for G predicts for two closed aspherical manifolds M and N with $\pi_1(M) \cong \pi_1(N) \cong G$ that any homotopy equivalence $M \to N$ is homotopic to a homeomorphism and in particular that M and N are homeomorphic.

Theorem (The Farrell-Jones Conjecture and the Borel Conjecture)

If G belongs to both $\mathcal{FJ}_K(\mathbb{Z})$ and $\mathcal{FJ}_L(\mathbb{Z})$, then the Borel Conjecture is true in dimension ≥ 5 and in dimension 4 if G is good in the sense of Freedman.

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The Novikov Conjecture for G predicts for a closed oriented manifold M together with a map $f \colon M \to BG$ that for any $x \in H^*(BG)$ the higher signature

$$\operatorname{sign}_x(M, f) := \langle \mathcal{L}(M) \cup f^*x, [M] \rangle$$

is an oriented homotopy invariant of (M, f)

Theorem (The Farrell-Jones, the Baum-Connes and the Novikov Conjecture)

If G belongs to $\mathcal{FJ}_L(\mathbb{Z})$ or to $\mathcal{BC}$, then the Novikov Conjecture holds for the group G.

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Theorem ($K_0(RG)$ and induction from finite subgroups)

Let R be a regular ring with $\mathbb{Q} \subseteq R$. Suppose $G \in \mathcal{FJ}(R)$. Then the map given by induction from finite subgroups of G

$$\operatorname*{colim}_{\mathrm{Or}_{\mathcal{FIN}}(G)} K_0(RH) \to K_0(RG)$$

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Conjecture (Bass Conjecture)

Let R be a commutative integral domain and let G be a group. Let $g \neq 1$ be an element in G. Suppose that either the order $|g|$ is infinite or that the order $|g|$ is finite and not invertible in R.
Then the Bass Conjecture predicts that for every finitely generated projective RG-module P the value of its Hattori-Stallings rank $\mathrm{HS}_{RG}(P)$ at (g) is trivial.

Theorem (The Farrell-Jones Conjecture and the Bass Conjecture)

Let G be a group. Suppose that $G \in \mathcal{FJ}(F)$ for every field F of prime characteristic.
Then the Bass Conjecture is satisfied for every integral domain R.

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Let R be a commutative integral domain and let G be a group. Let $g \neq 1$ be an element in G. Suppose that either the order $|g|$ is infinite or that the order $|g|$ is finite and not invertible in R.
Then the Bass Conjecture predicts that for every finitely generated projective RG-module P the value of its Hattori-Stallings rank $\mathrm{HS}_{RG}(P)$ at (g) is trivial.

Theorem (The Farrell-Jones Conjecture and the Bass Conjecture)

Let G be a group. Suppose that $G \in \mathcal{FJ}(F)$ for every field F of prime characteristic.
Then the Bass Conjecture is satisfied for every integral domain R.

Conjecture (Homotopy invariance of $L2$-torsion)

If X and Y are det-L^2-acyclic finite G-CW-complexes, which are G-homotopy equivalent, then their L^2-torsion agree:

$$\rho^{(2)}(X;\mathcal{N}(G)) = \rho^{(2)}(Y;\mathcal{N}(G)).$$

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Suppose that $G \in \mathcal{FJ}(\mathbb{Z})$. Then G satisfies the Conjecture above.

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Conjecture ((Stable) Gromov-Lawson-Rosenberg Conjecture)

Let M be a closed connected Spin*-manifold of dimension $n \geq 5$. Then $M \times B^k$ carries for some integer $k \geq 0$ a Riemannian metric with positive scalar curvature if and only if*

$$\mathrm{ind}_{C_r^*(\pi_1(M);\mathbb{R})}(M) \;=\; 0 \qquad \in KO_n(C_r^*(\pi_1(M);\mathbb{R})).$$

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Other versions of Isomorphism Conjectures

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Let $\mathcal{H}^?_*$ be an equivariant homology theory. It satisfies the Isomorphism Conjecture for the group G and the family $\mathcal{F}$ if the projection $E_{\mathcal{F}}(G) \to pt$ induces for all $n \in \mathbb{Z}$ a bijection

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Example

- The Farrell-Jones Conjecture for K-theory or L-theory respectively with coefficients in R is the Isomorphism Conjecture for $\mathcal{H}^?_* = H_*(-;\mathbf{K}_R)$ or $\mathcal{H}^?_* = H_*(-;\mathbf{L}_R^{\langle -\infty \rangle})$ respectively and $\mathcal{F} = \mathcal{VCYC}$.
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The rational version of the K-theoretic Farrell-Jones Conjecture for coefficients in $\mathbb{Z}$ is equivalent Farrell-Jones Conjecture for Pseudoisotopies.
In degree $n \leq 1$ this is even true integrally.

- Pseudo-isotopy and A-theory are important theories. In particular they are closely related to the space of selfhomeomorphisms and the space of selfdiffeomorphisms of closed manifolds.

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- The assembly map appearing in the Bost Conjecture

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- We discuss some relations between the Farrell-Jones Conjecture and the Baum-Connes Conjecture.
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Let G be a group. Let T be the set of conjugacy classes (g) of elements $g \in G$ of finite order.
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Theorem (Bartels-L.-Reich (2007), Bartels-Echterhoff-Reich (2007))

Let R be a ring. Then:

- *Every hyperbolic group and every virtually nilpotent group belongs to $\mathcal{FJ}(R)$;*
- *If G_1 and G_2 belong to $\mathcal{FJ}(R)$, then $G_1 \times G_2$ belongs to $\mathcal{FJ}(R)$;*
- *Let $\{G_i \mid i \in I\}$ be a directed system of groups (with not necessarily injective structure maps) such that $G_i \in \mathcal{FJ}(R)$ for $i \in I$. Then $\operatorname{colim}_{i \in I} G_i$ belongs to $\mathcal{FJ}(R)$;*
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- We emphasize that this result holds for all rings R. Actually we can even treat crossed product rings $R * G$. For more information about the last result and its proof we refer to the talks of Bartels.
- The groups above are certainly wild in Bridson's universe of groups.
- Many recent constructions of groups with exotic properties are given by colimits of directed systems of hyperbolic groups. Examples are.
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Definition (a-T-menable group)

A group G is *a-T-menable*, or, equivalently, has the *Haagerup property* if G admits a metrically proper isometric action on some affine Hilbert space.

- The class of a-T-menable groups is closed under taking subgroups, under extensions with finite quotients and under finite products.
  It is not closed under semi-direct products.
- Examples of a-T-menable groups are:
  - countable amenable groups;
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- A group G has *Kazhdan's property (T)* if, whenever it acts isometrically on some affine Hilbert space, it has a fixed point. An infinite a-T-menable group does not have property (T). Since $SL(n, \mathbb{Z})$ for $n \geq 3$ has property (T), it cannot be a-T-menable.

Theorem (Higson-Kasparov(2001))

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The Baum-Connes Conjecture is true for a certain class of groups which does contain some groups with property (T).

Theorem (Mineyev-Yu (2002))

The Baum-Connes Conjecture is true for subgroups of hyperbolic groups.

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- The Baum-Connes Conjecture and the Farrell-Jones Conjecture are not known for $SL_n(\mathbb{Z})$ for $n \geq 3$, mapping class groups and $\mathrm{Out}(F_n)$;
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- There seems to be no promising candidate of a group G which is a potential counterexample to the K- or L-theoretic Farrell-Jones Conjecture or the Bost Conjecture.
- The Baum-Connes Conjecture is the one for which it is most likely that there may exist a counterexample.
  One reason is the existence of counterexamples to the version with coefficients.
  Another reason is that $K_n(C_r^*(G))$ has certain failures concerning functoriality which do not exists for $K_n^G(\underline{E}G)$.
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  These failures are not present for $K_n(RG)$, $L^{\langle -\infty \rangle}(RG)$ and $K_n(l^1(G))$.

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- Most of the proofs of the Farrell-Jones Conjecture use methods from controlled topology.
- Roughly speaking, controlled topology means that one considers free modules with a basis and thinks of these basis elements as sitting in a metric space.
  Then a map between such modules can be visualized by arrows between these basis elements.
  Control means that these arrows are small.
- Our homological approach to the assembly map is good for structural investigations but not for proofs.
  For proofs of these Conjectures it is often helpful to get some geometric input.
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  It is of homotopy theoretic nature.
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- The methods of proof for the Baum-Connes Conjecture are of analytic nature.
  The most prominent one is the Dirac-Dual-Dirac method based on KK-theory due to Kasparov.
  KK-theory is a bivariant theory together with a product.
  The assembly map is given by multiplying with a certain element in a certain KK-group.
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